First Name: ____________________ Last Name: ________________________ Section: ______

Physics 207 Midterm 1

Wed., Feb. 20, 2008

Do not open the exam until you are instructed to start!

Section numbers: Gao (602 TR 2:25; 607 TR 4:35), McCaskey (603 TR 3:30; 608 TR 7:45); Wang (608 TR 7:45); Zhang (604 WF 1:20; 609 TR 9:55)

Instructions:

(1) **Do not open the exam until you are instructed to start!**
(2) Draw a box around your final answer for each part of the exam.
(3) Write your name at the top of each page.
(4) Note that each problem is worth 25 points.
(5) In most of the problems, answers are to be given symbolically (i.e. in terms of variables instead of numbers). In all of the problems, the letter g can be used to denote the magnitude of the gravitational force field (e.g. $g = 9.81\text{m/s}^2$).
(6) Anyone caught cheating will receive a 0 for the exam.

Tips:

(1) Relax and think calmly.
(2) There is no partial credit for multiple choice problems.
(3) Show work to receive at least some partial credit for non-multiple choice problems even if you do not get the problem completely correct. The graders have been instructed to be generous with giving partial credit points. Logic is at least as important as the answer.
(4) Suppose a problem has parts a) and b) and you need the answer to part a) to solve part b). If you have no idea how to do part a) but know how to do part b), then when you are doing part b), define clearly a variable that you would have obtained from part a) and use that variable for your final answer to part b).
(5) Write legibly and explain answers clearly. If the grader cannot understand your answer, it will be marked as incorrect.

(Do not write below)

Problem 1: ____________

Problem 2: ____________

Problem 3: ____________

Problem 4: ____________

Total:____________

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1. Suppose a one-dimensional motion of an object with mass m is found to have its acceleration given by $a(t) = \frac{c}{t^3}$ for $t > 1$ sec where c is a constant.

a) What is the velocity $v(t)$ if initially at $t = t_0$ (where t_0 is a constant greater than 1 sec) , we have $v(t_0) = v_0$? [Express your answer in terms of t, v_0, c, and t_0.] (15 pts)

answer————————————————————————————:

Velocity is obtained from the acceleration by an integral over time:

$$\begin{aligned} v(t) &= v_0 + \int_{t_0}^{t} dt' v(t') \\ &= v_0 + c\int_{t_0}^{t} \frac{dt'}{t'^3} \\ &= \boxed{v_0 - \frac{c}{2}(\frac{1}{t^2} - \frac{1}{t_0^2})} \end{aligned}$$

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b) Suppose you are told that the object reaches $x = x_f$ at time $t = 2t_0$. What is the initial position at time $t = t_0$? [Express your answer in terms of x_f, v_0, c, and t_0.] (10 pts)

answer————————————————————————————————————:

Integrating the velocity, one obtains the position to be

$$\begin{aligned} x(t) &= \int_{t_0}^{t} v(t')dt' \\ &= x_0 + v_0(t - t_0) - \frac{c}{2}(\frac{1}{t_0} - \frac{1}{t} - \frac{t - t_0}{t_0^2}) \\ &= x_0 + v_0(t - t_0) - \frac{c}{2}(\frac{2}{t_0} - \frac{1}{t} - \frac{t}{t_0^2}). \end{aligned}$$

Hence, we find

$$\begin{aligned} x_f &= x_0 + v_0(2t_0 - t_0) - \frac{c}{2}(\frac{2}{t_0} - \frac{1}{2t_0} - \frac{2t_0}{t_0^2}) \\ &= x_0 + v_0 t_0 - \frac{c}{4t_0} \end{aligned}$$

or

$$\boxed{x_0 = x_f - v_0 t_0 + \frac{c}{4t_0}}$$

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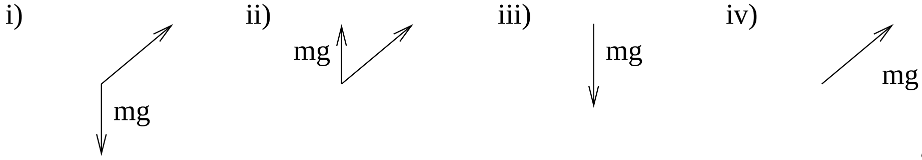
2. Multiple choice questions (circle the correct answer): Suppose a cruise ship is moving east at a constant velocity $v_{cw}\hat{\imath}$ with respect to the water which is flowing east with respect to the ground at constant velocity $v_{wg}\hat{\imath}$. [Assume that the ground is an inertial frame of reference as usual.]

a) On the deck of the cruise ship, suppose a basketball of mass m is launched eastward and upward with an initial velocity vector $\vec{v} = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath}$. Draw the free body force diagram of the basketball when the basketball has reached maximum height (i.e. maximum coordinate in the $\hat{\jmath}$ direction) and label the magnitude of the net force. [The rightward direction is $\hat{\imath}$ and the upward direction is $\hat{\jmath}$.] (5 pt)



answer is iii)

b) How long does it take for the ball to reach the maximum height after the launch? (5 pt)

$$i)\ \Delta t = v_{0y}/g$$

c) What is the maximum vertical displacement after the time of launch (i.e. maximum height minus the starting height)? (5 pt)

$$iii)\ \Delta y = \frac{1}{2}\frac{v_{0y}^2}{g}$$

d) How far east (i.e. $\hat{\imath}$ direction) did the basketball travel in the deck frame between the time of launch and the time of maximum height? (5 pt)

$iv)$ none of these

e) How far east (i.e. $\hat{\imath}$ direction) did the basketball travel in the ground frame between the time of launch and the time of maximum height? (5 pt)

$v)$ none of these

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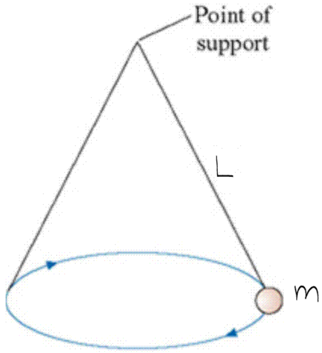
3) A conical pendulum is formed by attaching a ball of mass m to a string of length L, then allowing the mass to move in a horizontal circle.



a) Suppose the string is known to break if the tension exceeds T_c. What is the minimum angular speed (e.g. ω_{min} which is typically in radians per sec) for which the string will break? [The answer may involve at most $\{m, L, T_c\}$.] (10 pt)

answer———————————————————————————————:

Since, we have

$$T_c \sin\theta = T_c \frac{R}{L} = mR\omega_{\text{min}}^2,$$

we find

$$\boxed{\omega_{\text{min}} = \sqrt{\frac{T_c}{mL}}}$$

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b) When the string breaks, how fast is it travelling? (10 pt)

answer———————————————————————————————:

When the string breaks, the speed is

$$v = R\omega_{\text{min}}.$$

We can solve for R by balancing the vertical force:

$$T_c \cos\theta - mg = 0.$$

Since $\cos\theta = \frac{\sqrt{L^2 - R^2}}{L}$, we have

$$T_c \frac{\sqrt{L^2 - R^2}}{L} = mg$$

$$(mgL)^2 / T_c^2 = L^2 - R^2$$

$$R = \sqrt{L^2 - \frac{(mgL)^2}{T_c^2}} = L\sqrt{1 - \frac{(mg)^2}{T_c^2}}.$$

This means

$$v = L\sqrt{1 - \frac{(mg)^2}{T_c^2}}\omega_{\text{min}} = \boxed{\sqrt{1 - \frac{(mg)^2}{T_c^2}}\sqrt{\frac{T_c L}{m}}.}$$

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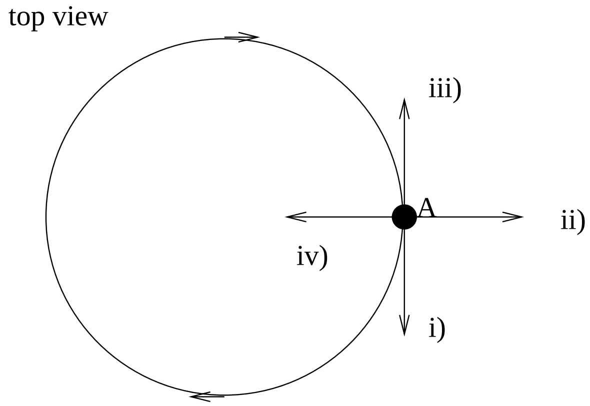
c) If the string breaks when the ball is at point A shown in the figure (the mass is rotating clockwise from this top view), which direction will the ball travel afterwards? (circle i, ii, iii, or iv) (5 pt)



Answer is i) because of the property of the velocity for a circular motion.

(extra work space for previous parts)

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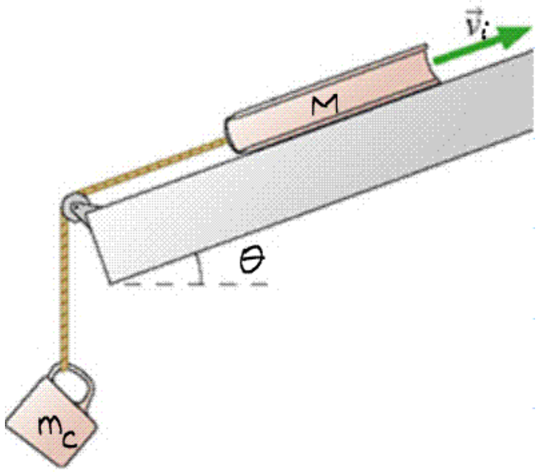
4) A physics book of mass M is connected by a string over a pulley to a dangling coffee cup of mass m_c. The book is given a push up the inclined plane making an angle θ with respect to the horizontal and released with a speed of v_i. The coefficient of kinetic friction is μ_k.



How far up the plane does the book slide after being released before coming to at least a momentary stop? [Express your answer in terms of $\{v_i, M, m_c, g, \theta, \mu_k\}$.] (25 pt)

answer————————————————————————————————————**-:**

As far as the books motion parallel to the plane is concerned, we have

$$-N\mu_k - T - Mg\sin\theta = Ma$$

Normal to the plane, we have

$$N - Mg\cos\theta = 0.$$

Finally, the cup only has a vertical motion, which means

$$T - m_c g = m_c a$$

where we have set the two accelerations equal because of the constant length of the string. Combining these equations, we can eliminate T and N to find

$$-Mg\cos\theta\mu_k - (m_c a + m_c g) - Mg\sin\theta = Ma$$

$$-Mg\cos\theta\mu_k - m_c g - Mg\sin\theta = (M + m_c)a$$

$$a = \frac{-Mg\cos\theta\mu_k - m_c g - Mg\sin\theta}{M + m_c}$$

Constant acceleration kinematics give

$$\boxed{\Delta x = -\frac{v_i^2}{2a} = \frac{v_i^2}{2}\frac{M + m_c}{Mg\cos\theta\mu_k + m_c g + Mg\sin\theta}}$$

(extra space for work)