Summary sheet for Lecture 10

Here I summarize some of the main results of lecture 10:

Suppose each measurement has a probability of giving a data value x according to the probability function $P(x)$. Then if there are N independent measurements, the probability of obtaining the data set $\{x_1, x_2, ..., x_N\}$ is

$$\mathcal{L} dx_1 dx_2 ... dx_N \equiv P(x_1)P(x_2)...P(x_N).$$

$\mathcal{L}$ is called the likelihood function.

Maximum likelihood principle: The experimental outcome is likely to be thmost probable outcome.

Example: If $P(x)$ for each measurement is Gaussian, we have the likelihood function

$$\mathcal{L} = (\frac{1}{\sqrt{2\pi\sigma_1^2}})^N \exp(-\sum_i^N \frac{(x_i - x_{true})^2}{2\sigma_1^2})$$

where x_{true} and σ_1 are two parameters defining $P(x)$. Maximizing $\mathcal{L}$ corresponds to setting partial derivatives with respect to x_{true} and σ_1 to zero and solving for x_{true} and σ_1. In this way, one obtains the maximum likelihood estimates of the theoretical parameters based on sample data as

$$x_{true} = \frac{1}{N}\sum_{i=1}^N x_i$$

$$\sigma_1^2 = \frac{1}{N}\sum_{i=1}^N (x_i - x_{true})^2$$

As explained in the appendix to lecture 10, if one wants the estimate to match the ideal theoretical distribution, a better estimate for the parameter σ_1 is

$$\sigma_1^2 = \frac{1}{N-1}\sum_{i=1}^N (x_i - x_{true})^2.$$

If $f(p_1, p_2, ..., p_\nu)$ is a function of ν parameters $\{p_1, p_2, ..., p_\nu\}$, and experimental work results in the extraction of $\{p_1, p_2, ..., p_\nu\}$ as

$$\{\bar{p}_1 \pm \sigma_{p_1}, \bar{p}_2 \pm \sigma_{p_2}, ..., \bar{p}_\nu \pm \sigma_{p_\nu}\},$$

the uncertainty in the function $f(p_1, p_2, ..., p_\nu)$ is

$$\sigma_f = \sqrt{\sigma_{p_1}^2 (\frac{\partial f}{\partial p_1}|_{\{p_{i\neq 1} = \bar{p}_{i\neq 1}\}})^2 + \sigma_{p_2}^2 (\frac{\partial f}{\partial p_2}|_{\{p_{i\neq 2} = \bar{p}_{i\neq 2}\}})^2 + ...}$$

In this way, you can get uncertainties in the derived quantities. You can also use this method to show that if one makes N measurements of quantity x where each measurement has an uncertainty of σ_x, the uncertainty for the average

$$\bar{x} \equiv \frac{1}{N}\sum_{i=1}^N x_i$$

is

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

which shows that if N is large, uncertainty is reduced.