Name Key

Exam #3

Physics 247

December 3, 2003

Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

1. Anti-deuterons are produced in the reaction $p + p \rightarrow d + \bar{d} + p + p$. The rest energies of the particles are (approximately) $m_p c^2 = 1$ GeV, $m_d c^2 = 2$ GeV.

   (a) What is the minimum center-of-mass energy required to produce anti-deuterons?

   $$(2 + 2 + 1 + 1)\ \text{GeV} = \boxed{6\ \text{GeV}}$$

   (b) If one of the original protons is at rest, what laboratory kinetic energy is required for the other proton?

   in GeV

   $$E_{cm}^2 = 36\ \text{GeV}^2 = (E+1)^2 - p^2c^2$$

   $$= E^2 + 2E + 1 - (E^2 - 1)$$

   $$= 2E + 2$$

   $$\therefore \quad E = 17\ \text{GeV}$$

   $$\Rightarrow \boxed{T = 16\ \text{GeV}}$$

2. A particle of mass m moving with relativistic velocity v strikes another particle, also of mass m, at rest, and sticks.

   (a) (10 pts) Write down conservation of energy and momentum for this situation, making sure you clearly define your symbols.

$$E_1 + E_2 = E_3 \qquad \gamma_1 m c^2 + m c^2 = \gamma_3 m' c^2$$

$$P_1 = P_3 \qquad \gamma_1 m v_1 = \gamma_3 m' v'$$

   (b) (15 pts) What is the mass of the new composite particle, in terms of v (and/or $\gamma(v)$) and m?

$$E_1^2 + 2E_1 E_2 + E_2^2 = E_3^2$$

$$P_1^2 = P_3^2$$

$$\underbrace{E_1^2 - P_1^2}_{m^2} + 2E_1 m + m^2 = \underbrace{E_3^2 - P_3^2}_{m'^2}$$

$$\therefore \quad m'^2 = 2m^2 + 2\gamma m^2$$

$$\therefore \quad \boxed{m' = m\sqrt{2\gamma + 2}}$$

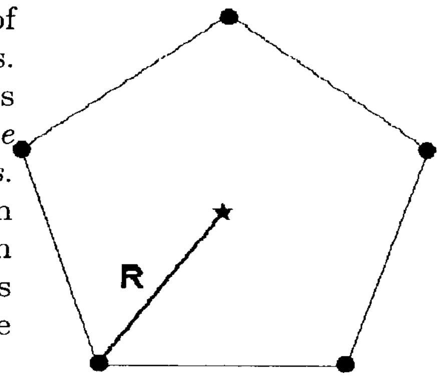
or,

$$(E_1 + Mc^2)^2 - p_1^2 = M'^2 c^4$$

$$2E_1 Mc^2 + 2M^2 c^4 = M'^2 c^4$$

$$M'^2 = (2\gamma + 2) M^2$$

3. Five balls, each of mass m, are arranged at the vertices of a regular pentagon and connected by massless, rigid rods. Assume the balls are small (i.e., point-like). The system is rotated with angular velocity ω about an axis *normal to the plane of the pentagon and passing through one of the balls.* What are the kinetic energy and the angular momentum in this system in terms of ω, m, and the radius R defined in the drawing? Hint: you may want to use the parallel axis theorem. The star denotes the center of the pentagon (there is no mass there).

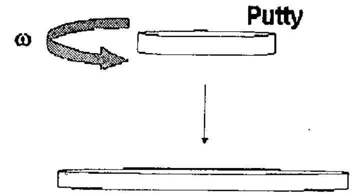


$$I_{com} = 5mR^2$$

$$I_{axis} = 5mR^2 + 5mR^2 = 10mR^2$$

$$\boxed{\begin{aligned} T &= \tfrac{1}{2} I_{axis}\,\omega^2 = 5mR^2\omega^2 \\ L &= I_{axis}\,\omega = 10mR^2\omega \end{aligned}}$$

4. A piece of putty, spinning with angular velocity ω about its center of mass, is launched towards a stationary disc with moment of inertia I_d, as shown. The putty sticks to the disc. Before the collision the moment of inertia of the putty is I_{pi}, and after the collision the moment of inertia of the putty is I_{pf}. The putty and the disc stick together, so obviously the total moment of inertia of the system after the collision is $I_d + I_{pf}$.



Disc, which is initially stationary.

18 pts (a) What is the final angular velocity of the disc-putty combination after the collision?

$$L_i = L_f$$

$$I_{pi}\,\omega = (I_d + I_{pf})\,\omega'$$

$$\boxed{\omega' = \frac{I_{pi}}{I_d + I_{pf}}\,\omega}$$

5 pts (b) Find the value of the integral

$$\int_{t_1}^{t_2} \tau_{\text{disk}} dt$$

where t_1 and t_2 are the beginning and ending times for the putty colliding with the disc, and τ_{disc} is the torque the putty applies to the disc as a function of time.

$$\int_{t_1}^{t_2} \tau_{disk}\,dt = \Delta L_{disk} = I_d\,\omega'$$

$$= \boxed{\frac{I_d\,I_{pi}}{I_d + I_{pf}}\,\omega} .$$

2 pts (c) Why might I_{pi} be different from I_{pf}?

Putty squishes.