

Physics 406 Exam 1

1. (30 pts) Given a 4-vector

$$v^\alpha = (1, 1, 1, 1)$$

- a) (10 pt) Compute what it looks like when boosted along the y-axis.
 b) (10 pt) Is v^α spacelike, null, or timelike?
 c) (10 pt) Compute what v_α looks like under the same boost as in a).

2. (20 pt) Suppose a space traveller's trajectory in an inertial frame is given by

$$\vec{x} = (R \sin(wt^2), R(1 - \cos(wt^2)), 0)$$

for $4R^2w^2t^2 < 1$ and $8\pi wR^2 < 1$. Suppose the traveller's wrist watch (given by the variable τ) registers $\tau = 0$ at $t = 0$. What does it register after one round trip? (You may leave the answer in terms of a definite integral without computing the integral.)

3. (30 pt) Particle A with mass m_A is at rest in an inertial frame. Suppose it decays to two particles (both called B) each with mass m_B .
 a) (10 pt) Compute the speed of the B particle.
 b) (10 pt) Compute the magnitude of the momentum of the B particle.
 c) (10 pt) Suppose one of the B emission directions makes an angle $\bar{\theta}$ with respect to the x-axis in the rest frame of particle A. What is the angle θ that the same B particle emission direction makes with respect to the x-axis in the frame in which the particle A is moving along the x-axis with a speed u (you may express your answer in terms of $|\vec{p}|$ representing the answer to part b)?

4. (20 pt) Suppose you are given a solution to the wave equation

$$[\partial_t^2 - \partial_x^2]\phi(t, x) = 0.$$

[You have seen in class that the operator to the equation above is Lorentz invariant $(t, x) \rightarrow (\gamma t + \gamma v x, \gamma x + \gamma v t)$.] Suppose you are given that ϕ satisfies the equation

$$\partial_x \partial_t \phi(t, x) + \frac{v}{2} \partial_x^2 \phi(t, x) = f(t, x),$$

evaluate

$$Q \equiv [\partial_t^2 - \partial_x^2]\phi(t, x + vt)$$

which you should recognize as the Galilean transformation of ϕ . Express your answer in terms of f and v .