Physics 406 Exam 2

Thursday, 11-13-03, 1:20-2:10 PM

As usual we are working in units with $c = G_N = 1$.

1. (25 pts) Suppose a point-like test mass (of mass q) is released from rest a distance d away from a center of a fixed spherical body which has radius R and mass $M \ll R$. Assume that the spherical body is located at the origin and that $d \gg R$.

a) (3 pt) Write down the acceleration of the test mass as a function of r using Newton's Law.

b) (10 pt) Write down the geodesic equation for the test mass position $x^\mu(\tau)$ where τ is the proper time (leave the answer in terms of $\Gamma^\mu{}_{\alpha\beta}$).

c) (12 pt) Assuming that $\frac{dx^0}{d\tau} = 1$, $r > R$ (and non-relativistic speed approximation), compute the Christoffel symbol component Γ^r_{tt} (i.e. Γ^1_{00} in the (t, r, θ, ϕ) coordinates).

2. (25 pts) Suppose you are given that at point P in the coordinate system $x^\mu = (t, x, y, z)$ that

$$\frac{\partial V^\lambda}{\partial x^\mu}|_P = 0$$

and

$$V^\lambda|_P = (1, 0, 0, 0)$$

where V^λ is a covariant vector under general coordinate transformations. Compute

$$\frac{\partial \bar{V}^0}{\partial \bar{x}^0}|_P$$

(at point P) in the coordinate system $\bar{x}^\mu = (u, v, y, z)$ where

$$u = \sqrt{t} - x$$

$$v = \sqrt{t} + x$$

assuming that the point P is located at $(t, x, y, z) = (1, 0, 0, 0)$. Hint: Either remember the inhomogeneous transformation rule of $\partial_\mu V^\lambda$ or even more simply start from the definition of how V^λ transforms under coordinate transformation.

3. (30 pts) Suppose on a two dimensional surface parameterized by (x, y) (valid for $x \neq 0$) with a metric

$$ds^2 = \frac{dx^2}{Q^2} + x^2 dy^2,$$

there is a curve C parameterized by $(x = x_0, y)$ where y runs from 0 to 1. (Here, Q is a constant.) If there is a vector $\vec{V}$ on the curve at position $(x = x_0, y = 0)$ with magnitude $\vec{V}(x = x_0, y = 0) = \hat{y}$ ($\hat{y}$ is the usual basis unit vector), what is the value of the vector when it is parallel transported along the curve C to $(x = x_0, y = 1)$? (Hint: $\Gamma^2_{2\beta} = \frac{1}{x}\delta_{\beta 1}$ where the notation is as usual, i.e. $x^1 = x$ and $x^2 = y$. Hence, only $\Gamma^1_{2\beta}$ needs to be computed.)

4. (20 pts) Starting from the Bianchi identities

$$R_{\alpha\beta\mu\nu;\lambda} + R_{\alpha\beta\lambda\mu;\nu} + R_{\alpha\beta\nu\lambda;\mu} = 0$$

compute

$$R^\mu{}_{023;\mu}$$

in terms of

$$f_1 \equiv R_{03;2}$$

$$f_2 \equiv R_{20;3}.$$