PHYSICS 406: PROBLEM SET 4

due:: Thursday, March 9, 2006

Problems

(1) Show that
$$T^{\mu}_{\ \sigma;\nu} = T^{\mu}_{\ \sigma,\nu} + \Gamma^{\mu}_{\nu\lambda} T^{\lambda}_{\ \sigma} - \Gamma^{\lambda}_{\nu\sigma} T^{\mu}_{\ \lambda}$$
is a tensor.

(2) Using the definition
$$g_{\mu\nu} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} \frac{\partial \xi^{\beta}}{\partial x^{\nu}} \eta_{\alpha\beta}$$
show that
$$g_{\mu\nu;\lambda} = 0.$$

(3) For a 2-dimensional flat, Euclidean space described by polar coordinates (r, θ), assume that geodesics are the usual straight lines.

(a) Find the connection coefficients $\Gamma^{\alpha}_{\beta\gamma}$ using your knowledge of the straight line geodesics and the geodesic equation
$$\frac{d^2 x^{\mu}}{d\lambda^2} + \frac{dx^{\alpha}}{d\lambda} \frac{dx^{\beta}}{d\lambda} \Gamma^{\mu}_{\alpha\beta} = 0$$

(b) Next, in the cartesian coordinates (x, y) which are related to the (r, θ) coordinates in the usual way, take the Christoffel symbols to be 0. Using the transformation law for the connection coefficients, find the connection coefficients in the (r, θ) coordinates.

(c) Finally, from the line element
$$ds^2 = dr^2 + r^2 d\theta^2$$
find the Christoffel symbols, in the usual way, as derivatives of the metric coefficients.

(4) Compute the covariant derivatives of the following (express in terms of the usual derivatives and the Christoffel symbol)

(a) $T^{\alpha\beta}_{\ \ \gamma\delta}$

(b) $T^{\mu\nu} V_{\mu} V_{\nu}$