Lec 3: Conclusion of Hints from EM and light

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1 Failure of Galilean Transformation

We saw in the previous lecture that the potential due to a moving charge looks like

$$\phi = \frac{q\gamma}{4\pi} \frac{1}{\sqrt{(\gamma x - \gamma v t)^2 + y^2 + z^2}}.$$

This is a bit surprising since we naively expected to see

$$\phi = \frac{q}{4\pi} \frac{1}{\sqrt{(x - vt)^2 + y^2 + z^2}}$$

based on Galilean boost ($\vec{x} \to \vec{x} - \vec{v}t$) of Coulomb's Law. What is the origin of this surprise? The problem is that unlike Newton's second laws, Maxwell equations are not invariant under the Galilean transformations.

To see another example of the fact that Maxwell equations not obeying Galilean relativity, consider the equation from lecture 1 describing light propagation:

$$[\partial_k \partial^k - \partial_t^2]\vec{E} = 0 \tag{1}$$

Now, consider the Galilean boost:

$$t \to t$$

$$\vec{x} \to \vec{x} - \vec{v}t$$

or equivalently

$$t = t'$$

$$\vec{x} = \vec{x}' - \vec{v}t'$$

where $\vec{x}'$ is the coordinate in the boosted frame. Now, note that

$$\begin{aligned}
\frac{\partial}{\partial t'}\vec{E}(t(t',\vec{x}'),\vec{x}(t',\vec{x}')) &= \frac{\partial t}{\partial t'}\frac{\partial}{\partial t}\vec{E}(t,\vec{x}) + \frac{\partial x^i}{\partial t'}\frac{\partial}{\partial x^i}\vec{E}(t,\vec{x}) \\
&= \frac{\partial}{\partial t}\vec{E}(t,\vec{x}) + \frac{\partial x^i}{\partial t'}\frac{\partial}{\partial x^i}\vec{E}(t,\vec{x}) \\
&= \frac{\partial}{\partial t}\vec{E}(t,\vec{x}) - v^i\frac{\partial}{\partial x^i}\vec{E}(t,\vec{x})
\end{aligned}$$

Note that this means

$$\partial_t = \partial_{t'} + v^i \frac{\partial}{\partial x^i}$$

Exercise

Compute $\frac{\partial}{\partial x'^i} \vec{E}(t(t', \vec{x}'), \vec{x}(t', \vec{x}'))$ in terms of derivatives of $\vec{E}(t, \vec{x})$ with respect to t and x^j.

Hence, we have

$$\partial_t = \partial_{t'} + v^i \frac{\partial}{\partial x'^i}.$$

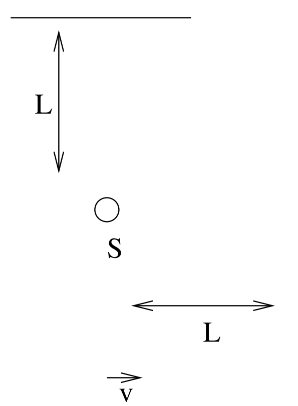
Using this formal operator, we finally can rewrite Eq. (1) as

$$[\partial_t^2 - \partial_j^2]\vec{E} = [\partial_{t'}^2 - (\frac{\partial}{\partial x'^j})^2]\vec{E} + [2v^i \frac{\partial}{\partial x'^i}\partial_{t'} + (v^i \frac{\partial}{\partial x'^i})(v^j \frac{\partial}{\partial x'^j})]\vec{E} = 0$$

which has a different form than Eq. (1) due to the last two terms.

2 Michelson-Morley Experiment

People used to think that ether was the medium through which light signals travelled much like sound waves in air. Consider two mirrors shown below (one mirror is positioned at distance L at right angles with respect to the light source point S) where the system is moving with respect to ether with velocity $\vec{v}$.



If the velocities add, just like Galilean relativity, the round trip time of light parallel to $\vec{v}$ is

$$\Delta\tau_{\text{parallel}} = \frac{2L}{1 - v^2}$$

while the round trip time light perpendicular to $\vec{v}$ is

$$\Delta\tau_{\text{perp}} = \frac{2L}{\sqrt{1 - v^2}}$$

Hence, there is a difference in light travel time that can be measured (using interferometric technique, for example). However, no such time difference has been measured. Indeed, experimentally, light speed seems to be constant in all frames of reference.