Lec 6: Tensors in Special Relativity

7th February 2006

This lecture is taken mostly from chapter 10.

1. Tensors are mathematical objects having definite transformation properties under coordinate transformations. (pg. 11 of the book)

2. Special relativistic tensors transform under coordinate transformations according to the Lorentz transformations.

3. Example: Electromagnetic tensor of Maxwell equations.

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

$$\tilde{F}_{\mu\nu} \equiv \frac{-1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta} = \begin{pmatrix} 0 & -B_1 & -B_2 & -B_3 \\ B_1 & 0 & -E_3 & E_2 \\ B_2 & E_3 & 0 & -E_1 \\ B_3 & -E_2 & E_1 & 0 \end{pmatrix}$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the Minkowski space analog of ϵ_{ijk} that we learned before.

Exercise

Compute $F^{\mu\nu}$

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4. Maxwell Equations

$$\partial_\mu F^{\mu\nu} = -j^\nu$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

Exercise

Which Maxwell equation does the following correspond to?

$$\partial_\mu F^{\mu 0} = -j^0$$

Exercise

Suppose there is a point charge q sitting at the origin in the lab frame. Suppose you view the field of the charge from another inertial frame moving at speed v in the $\hat{x}$ axis. What is the magnetic field that you observe?