Physics 721 Homework 3
Assigned: Tues. 2/1/05
Due: Mon. 2/7/05

1. A point charge e of mass m is moving in the field of a static point charge $-e$. The Lagrangian in cylindrical coordinates (r, ϕ) is

$$L = -m\sqrt{1 - (\dot{r}^2 + r^2\dot{\phi}^2)} + \frac{e^2}{r}$$

where the action is given by

$$S = \int dt L(r, \phi, \dot{r}, \dot{\phi})$$

with t ordinary time.

a) Show that the angular momentum

$$K \equiv \frac{\partial L}{\partial \dot{\phi}}$$

and the Hamiltonian are conserved.

b) Using part a), derive the first order differential equation for $r(t)$.

c) Derive the Keplerian relationship between the radius and period for circular orbits (obtain a relationship between the radius R and period T).

2. Jackson 12.14

3. Solve the equation

$$\vec{\nabla}^2 G(\vec{x}, \vec{y}) = \delta^{(3)}(\vec{x} - \vec{y})$$

for $G(\vec{x}, \vec{y})$ using the Fourier transform method where $\vec{\nabla}^2 \equiv \sum_{i=1}^{3} \partial_i^2$ and the boundary condition is that $G(\vec{x}, \vec{y}) = 0$ for $|\vec{x}| \to \infty$. An integral that will be useful is

$$\int_0^\infty \frac{dq}{q} \sin q = \frac{\pi}{2}$$

which can be proven through the contour integration method. (You do not need to prove the integral.)

4. Suppose two charges q_1 and q_2 are moving with the same constant relativistic velocity $\vec{v}$ but are separated by a spatial vector $\vec{R}$ (the direction is from q_2 to q_1) and $\vec{R} \cdot \vec{v} = |\vec{R}||\vec{v}| \cos\theta$. What is the force on q_1?

5. How long did the homework take?