Lecture 12 (2/14/05)

Electrostatics

1. Electrostatics $\to$ relativistic Green's function with time integrated over

2. Electric field "boundary conditions" (Jackson 1.6)

$$\vec{n}_+ \cdot (\vec{E}_+ - \vec{E}_-) = \sigma$$

electrostatic:

$$\vec{t}_+ \cdot (\vec{E}_+ - \vec{E}_-) = 0$$

3. Potential boundary cond.

$$\Phi_+ = \Phi_-$$

$$-\frac{\partial \Phi}{\partial n}|_+ - (-\frac{\partial \Phi}{\partial n}|_-) = \sigma$$

4. Boundary conditions of electrostatic potentials (Jackson 1.8-1.10)

Green's second identity:

$$\int_V d^3x(\phi \nabla^2 \psi - \psi \nabla^2 \phi) = \oint_S da[\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n}]$$

Dirichlet:

$$\Phi(\vec{x}) = \frac{1}{4\pi} \int_V d^3x' G_D(\vec{x}, \vec{x}') \rho(\vec{x}') - \frac{1}{4\pi} \oint_S da' \Phi(\vec{x}') \frac{\partial G_D}{\partial n'}$$

Neumann:

$$\Phi(\vec{x}) = \langle \Phi \rangle_S + \frac{1}{4\pi} \int_V d^3x' G_N(\vec{x}, \vec{x}') \rho(\vec{x}') + \frac{1}{4\pi} \oint_S da' \frac{\partial \Phi}{\partial n'} G_N(\vec{x}, \vec{x}')$$