Lecture 17 (2/25/05)

Electrostatics

14. Laplace equations in various coordinates

$$\nabla^2 = \sum_{ij} \frac{1}{\sqrt{g}} \partial_i [\sqrt{g} g^{ij} \partial_j]$$

$$dl^2 = g_{ij} dx^i dx^j$$

$$\sqrt{g} = \sqrt{det[g_{ij}]}$$

$$\sum_i g^{ki} g_{ij} = \delta^k{}_j$$

15. Laplace equation in spherical form (Jackson 3.1) and separation of variables

16. Legendre equation and Legendre polynomials (Jackson 3.2).

17. Associated Legendre Functions and the Spherical Harmonics (Jackson 3.5)

$$\Phi(r,\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} [A_{lm} r^l + B_{lm} r^{-(l+1)}] Y_{lm}(\theta,\phi)$$

$$A_{lm} r^l + B_{lm} r^{-(l+1)} = \int d\Omega Y^*_{lm}(\theta,\phi) \Phi(r,\theta,\phi)$$

$$Y_{lm}(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P^l_m(\cos\theta) e^{im\phi}$$

$$P^l_m(x) = \frac{(-1)^m}{2^l l!} (1-x^2)^{m/2} \frac{d^{l+m}}{dx^{l+m}} (x^2-1)^l$$