Lecture 20 (3/4/05)

Electrostatics

22. More properties of Bessel functions

$$\int_a^b dx J_m(kx) J_m(lx) x = \frac{1}{k^2 - l^2}[lx J_m(kx) J'_m(lx) - kx J_m(lx) J'_m(kx)]\Big|_a^b$$

$$\int J_m^2(kx) x dx = \frac{1}{2}(x^2 - \frac{m^2}{k^2}) J_m^2(kx) + \frac{1}{2} x^2 [J'_m(kx)]^2$$

$$J_{m-1} = \frac{m}{x} J_m(x) + J'_m(x)$$

$$J_{m+1} = \frac{m}{x} J_m(x) - J'_m(x)$$

23. Boundary-value problems in cylindrical coordinates (Jackson 3.8)

24. Expansion of Green Functions in Spherical Coordinates (Jackson 3.9)

$$\nabla_x^2 G(\vec{x}, \vec{x}') = -\delta^{(3)}(\vec{x} - \vec{x}')$$

$$G(\vec{x}, \vec{x}') = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{Y_{lm}^*(\theta', \phi') Y_{lm}(\theta, \phi)}{(2l+1)[1 - (a/b)^{2l+1}]} (r_<^l - \frac{a^{2l+1}}{r_<^{l+1}})(\frac{1}{r_>^{l+1}} - \frac{r_>^l}{b^{2l+1}})$$