Lecture 21 (3/7/05)

Multipoles and Dielectrics

25. Multipole expansion

$$q_{lm} = \int d^3x' Y^*_{lm}(\theta', \phi') r'^l \rho(\vec{x}')$$

$$\Phi(\vec{r}) = \sum_{l,m} \frac{Y_{lm}}{2l+1} \frac{q_{lm}}{r^{l+1}}$$

Cartesian quadrupole moment:

$$Q_{ij} = \int d^3x' \rho(\vec{x}')(3x'_i x'_j - r'^2 \delta_{ij})$$

Relationship to spherical quadrupole moment: e.g.

$$q_{21} = \frac{-1}{3}\sqrt{\frac{15}{8\pi}}(Q_{13} - iQ_{23})$$

Spherical tensors:

$$Y_{lm}(\Omega_R) = \sum_{m'} D^l_{mm'}(R) Y_{lm'}(\Omega)$$

The value of q_{lm} for the lowest nonvanishing multipole moment of any charge distribution is independent of the choice of origin of the coordinates.