SUSY QCD phasing invariant example

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Consider first a toy model SUSY QCD with two gauged chiral multiplets $\Phi_1(\phi_1,\psi_1)$ and $\Phi_2(\phi_2,\psi_2)$ transforming under 3 and $\bar{3}$, respectively, of $SU(3)$. After putting the auxilliary fields on shell, we have

$$\begin{aligned} L &= \Phi_1^\dagger e^{2gV^a t^a}\Phi_1|_D + \Phi_2^\dagger e^{-2gV^a t^{a*}}\Phi_2|_D + \text{gauge kinetic} && (1)\\ &= (D_\mu\phi_1)^\dagger(D^\mu\phi_1) + i\psi_1\sigma^\mu D_\mu^\dagger\bar{\psi}_1 + i\sqrt{2}g(\phi_1^\dagger t^a\lambda^a\psi_1 - \bar{\psi}_1 t^a\bar{\lambda}^a\phi_1) + && (2)\\ &\quad (D_\mu^*\phi_2)^\dagger(D^{\mu*}\phi_2) + i\psi_2\sigma^\mu D_\mu^T\bar{\psi}_2 - i\sqrt{2}g(\phi_2^\dagger t^{a*}\lambda^a\psi_2 - \bar{\psi}_2 t^{a*}\bar{\lambda}^a\phi_2) - && (3)\\ &\quad -\frac{1}{2}g^2\sum_a(\phi_1^\dagger t^a\phi_1 - \phi_2^\dagger t^{a*}\phi_2)^2 + \text{gauge kinetic} && (4)\end{aligned}$$

where $D_\mu = \partial_\mu + igt^aA_\mu^a$, the vector multiplet is given by $V(A_\mu,\lambda)$, and "gauge kinetic" gives the kinetic energy terms for the vector multiplet (including the anomaly terms). We shall now consider CP violating phases, neglecting any anomaly contributions. We start by adding a SUSY mass term to this Lagrangian:

$$\Delta L_0 = -m\Phi_1^T\Phi_2|_F = |m|^2(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) - |m|(e^{i\rho}\psi_1\psi_2 + e^{-i\rho}\bar{\psi}_2\bar{\psi}_1) \tag{5}$$

where $e^{i\rho}$ is a phase of m. Note that this phase ρ is not physical since we can use

$$\begin{aligned}\psi_i &\to e^{-i\rho/2}\psi_i\\ \lambda_i &\to e^{i\rho/2}\lambda_i\end{aligned}$$

to eliminate it. Now, consider introducing a soft-susy-breaking mass term for the gluino:

$$\Delta L_1 = -|M_3|(e^{i\theta_\lambda}\lambda^a\lambda^a + e^{-i\theta_\lambda}\bar{\lambda}^a\bar{\lambda}^a) \tag{6}$$

where $e^{i\theta_\lambda}$ is an arbitrary phase of M_3. One can rotate away this phase by a phase redefinition

$$\begin{aligned}\lambda^a &\to e^{-i\theta_\lambda/2}\lambda^a && (7)\\ \phi_i &\to e^{-i\theta_\lambda/2}\phi_i. && (8)\end{aligned}$$

(One cannot use ψ rotation to eliminate θ_λ since that does not leave its mass term invariant.) We can similarly use

$$\begin{aligned}\psi_i &\to e^{-i\rho/2}\psi_i && (9)\\ \phi_i &\to e^{-i\rho/2}\phi_i && (10)\end{aligned}$$

to still elimate the quark mass phase ρ. However, suppose we introduced a soft-breaking term of the form

$$\Delta L_2 = |m_{LR}|(e^{i\phi}\phi_2^T\phi_1 + e^{-i\phi}\phi_1^\dagger\phi_2^*) \tag{11}$$

where ϕ is some arbitrary phase. Now, we find that Eq. (8) does not eliminate the original ΔL_1 phase. However, we do not have 3 phases, since by successive applications of Eq. (8) and Eq. (10), we can absorb θ_λ and ρ into ϕ by redefining

$$\phi \to \phi + \theta_\lambda + \rho$$

Hence, we have only one physical phase: i.e. $\phi - \theta_\lambda - \rho = \text{const} \equiv \phi_{phys}$.

In the way we have presented our results, it seems that only the introduction of ΔL_2 introduced a physical phase apart from the anomaly contribution in this toy model. However, this is merely an artifact of the order of our construction. For example, if ΔL_1 is missing, but only ΔL_0 and ΔL_2 exists, there still is no physical phase.