**Physics 449** **HOMEWORK SET 1** **Spring 2009**

**Due Wednesday January 28**

1) The spin operators for a particle with spin-$\frac{1}{2}$ are given in equations (10-4) and (10-5) of the text.

(a) Verify that the operators obey the usual commutation relations, $[S_x, S_y] = i\hbar S_z$, etc.

(b) Find the operator $S^2$.

2) (a) Find the eigenvalues and the normalized eigenvectors of the operator $S_x$.

(b) Imagine that an electron has a spin wave function $\chi = \frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2} \\ i\end{pmatrix}$. Find the probabilities that a measurement of $S_x$ would give values $\hbar/2$ and $-\hbar/2$

3) A review problem: The radial wave functions for states in hydrogen have a relatively simple form. In particular, the lowest state for each $l$ (i.e. the state with $n = l+1$) has the form

$$R(r) = N r^l e^{-r/na_0},$$

where $a_0$ is the Bohr radius. Find the expectation values of $1/r$ and of $1/r^2$ for these states.

4) Evaluate the relativistic correction to the kinetic energy,

$$\Delta E_{\text{rel}} = \langle\psi| - \frac{1}{8}\frac{p^4}{m^3c^2}|\psi\rangle$$

for the states considered in Problem 3 above. Your result should be consistent with the general formula

$$\Delta E_{\text{rel}} = \frac{1}{2}mc^2\alpha^4\frac{1}{n^3}\,[\frac{3}{4n} - \frac{2}{2\ell+1}\,].$$

[Hints: The states $\psi$ are eigenfunctions of our original nonrelativistic Hamiltonian $H$ so you can replace $\frac{p^2}{2m}\psi$ by $(H - V)\psi = (E_n - V)\psi$. Similarly, as we discussed in class, $(\frac{p^2}{2m})^2\psi$ can be replaced by $(E_n - V)^2\psi$. The resulting integrals can be evaluated using your results from Problem 3.]

5) In class we solved the time independent Schrodinger equation for an electron at rest in an external field $\vec{B} = B\hat{z}$. The energy operator for this situation is just $H = -\vec{\mu}\cdot\vec{B} = g\frac{eB}{2m}S_z$.

(a) Solve the time **dependent** Schrodinger equation $H\chi = i\hbar\frac{\partial}{\partial t}\chi$ starting from an arbitrary initial state $\chi(0) = \begin{pmatrix}a_0 \\ b_0\end{pmatrix}$.

(b) Use your time dependent wave function to compute $\langle S_x\rangle$ and $\langle S_y\rangle$ for the special case $a_0 = b_0 = \frac{1}{\sqrt{2}}$.

(c) Describe in words what is happening.