Physics 449 **HOMEWORK SET 10** **Spring 2009**

Due Wednesday May 6

36) A proton beam of intensity $0.15\,\mu$A is incident on a thin ^{90}Zr foil target. The target foil has a mass per unit area of $0.6\,\mathrm{mg/cm^2}$ and the atomic mass of ^{90}Zr is approximately 90 u. A 100% efficient detector $0.25\,\mathrm{mm}^2$ in area is located 20 cm from the target at $\theta = 45°$. Find the differential cross section in b/sr (1b = 1 "barn" $= 10^{-24}\,\mathrm{cm}^2$) if the measured counting rate is 420/s.

37) As we saw in class, one can expand a plane wave e^{ikz} in a "partial wave" angular momentum series of the form

$$e^{ikz} = \sum_\ell \psi_\ell$$

where $\psi_\ell = R_\ell(r) Y_\ell^0(\theta,\phi)$. The functions R_ℓ can be found by integrating Y_ℓ^{0*} times $e^{ikr\cos\theta}$ over θ and ϕ. Our result for $\ell = 0$ was $\psi_0 = \sin kr/kr$. Use this same procedure to determine the $\ell = 1$ partial wave ψ_1. Verify your result by comparing with Bauer's formula,

$$e^{ikz} = \sum_\ell (2\ell+1)\, i^\ell\, j_\ell(kr)\, P_\ell(\cos\theta),$$

where the j_ℓ functions are given on page 142 of the text.

38) (a) Determine the $\ell = 0$ phase shift for scattering of electrons of energy E = 5 eV from a square-well potential of depth $V_0 = 2$ eV and radius $a = 0.05$ nm. [Hint: Solve for $u(r)$ inside and outside the well and match the solutions. Remember that $u(r)$ must go to zero at $r = 0$.]

(b) Find the differential cross section (in b/sr) assuming that contributions from the partial waves with $\ell > 0$ can be neglected.

39) Find the differential cross section for scattering from a "perfectly rigid sphere" of radius a (i.e. a potential that goes to infinity for $r \le a$). Take $k = 0.1$/nm and $ka = 1/3$. Include both $\ell = 0$ and $\ell = 1$, but ignore contributions from higher ℓ-values. [Hints: The radial wave functions $u_\ell(r)$ must go to zero at $r = a$. For $r > a$ they are of the form $\alpha_\ell\, j_\ell(kr) + \beta_\ell\, n_\ell(kr)$. To find the phase shifts take the limit $r \to \infty$ using (8-65) and (8-66). The differential cross section will be of the form $\mathrm{A} + \mathrm{B}\cos\theta + C\cos^2\theta$.]

40) Starting from the Lorentz transformation (the formulas for z' and t' in terms of z and t) derive, by algebra, the inverse Lorentz transformation (formulas for z and t in terms of z' and t'). You should find that the formulas are identical except for the sign of v.

41) Let Δt and $\Delta t'$ stand for the time difference between two events as measured in S and S' respectively.

(a) Starting from the Lorentz transformation, find a formula for $\Delta t'$ in terms of Δt assuming the two events occur at the same place in S.

(b) Find a formula for $\Delta t'$ in terms of Δt assuming the two events occur at the same place in S'.

(c) Charged π-mesons at rest in the laboratory have a mean lifetime of 2.6×10^{-8} s. Find the mean lifetime of a beam of π's moving at a velocity of 0.99c.

42) Suppose A_μ and B_μ ($\mu = 1, 4$) are any 4-vectors. Show that $\Sigma_\mu A_\mu B_\mu$ is an invariant – *i.e.* a quantity that has the same value in all frames.