Physics 449 **HOMEWORK SET 2** **Spring 2009**

Due Wednesday February 4

6) Deuterium is an isotope of hydrogen in which the nucleus has spin $I=1$.

(a) Find the total number of 2p states in deuterium by taking into account all possible values of the quantum numbers m, m_s and m_I.

(b) Write down the possible values of j, and then for each j find the possible values of the quantum number f, where $\vec{F} \equiv \vec{J} + \vec{I}$.

(c) Count the number of states by listing all the possible values of the quantum numbers j, f and m_f. Your answer should agree with part (a).

7) In this problem we will construct some of the J^2, J_z eigenfunctions for the 2p state in hydrogen. The basis functions for the problem are the functions $Y_l^m \chi^{\pm}$. According to what we said in class, the state $|j,m\rangle = |\frac{3}{2},\frac{3}{2}\rangle$ is just $Y_1^1\chi^+$.

(a) Find the state $|\frac{3}{2},\frac{1}{2}\rangle$ by applying the lowering operator J_- to $|\frac{3}{2},\frac{3}{2}\rangle$. To find the state, write $J_- = L_- + S_-$ and then use Equations (7-20) and (7-24) of the text, together with the corresponding equations for S_- to decide what L_- and S_- do to $Y_1^1\chi^+$.

(b) Normalize the state you found in part (a) and then find $|\frac{1}{2},\frac{1}{2}\rangle$ by using the fact that $|\frac{1}{2},\frac{1}{2}\rangle$ and $|\frac{3}{2},\frac{1}{2}\rangle$ must be orthogonal.

8) Verify by brute force calculation that the states $|\frac{3}{2},\frac{1}{2}\rangle$ and $|\frac{1}{2},\frac{1}{2}\rangle$ found in Problem (7) actually are eigenstates of J^2. Here are some hints and suggestions. First, write J^2 in the form $J^2 = L^2 + S^2 + 2\vec{L}\cdot\vec{S}$. You can then save a lot of algebra by making use of the fact that Y_l^m and $\chi^{\pm}$ are eigenfunctions of L^2 and S^2 with known eigenvalues. Simply replace (for example) $L^2 Y_l^m$ by $l(l+1)\hbar^2 Y_l^m$. To complete the problem you will still need to work out the result of $\vec{L}\cdot\vec{S}$ acting on each term. Here the algebra simplifies if you start by showing that $\vec{L}\cdot\vec{S} = \frac{1}{2}L_+S_- + \frac{1}{2}L_-S_+ + L_zS_z$ (recall $L_{\pm} = L_x \pm iL_y$ etc.). Then use Equations (7-20), (7-23) and (7-24) to work out the results. Before getting too far along, think about how to organize the work so that you don't need to do the same long calculations for $|\frac{3}{2},\frac{1}{2}\rangle$ and then over again for $|\frac{1}{2},\frac{1}{2}\rangle$.

9) A particle of mass m in an infinite square well (that extends from 0 to L) is subject to a perturbation $H_1(x) = A\cos\frac{2\pi}{L}x$.

(a) Find the first-order shift of the ground state energy.

(b) Find a general formula for the first-order energy shift of the excited states, $n \geq 2$. Remember that the square well energy eigenstates are given by $\phi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x$.

(c) In first order, the wave functions have the form $\psi_n = \phi_n + \sum_k c_{nk}\phi_k$. Determine which of the c_{nk} coefficients will be non-zero for arbitraty n. [Hint: There is a trig identity for $\cos x \sin y$ that will let you write $H_1\phi_n$ as an expansion over the energy eigenstates.]

10) A charged particle is confined in a one-dimensional harmonic oscillator potential. Suppose we turn on a weak electric field (E) so that the potential is shifted by an amount $H_1 = -qEx$.

(a) Show that the first order energy shift is zero for all energy levels. [Hint: Remember about raising and lowering operators for the harmonic oscillator. See Section 6-2 of the text.]

(b) Calculate the second order energy shift of the ground state.

11) The energy eigenfunctions for the 2-dimensional harmonic oscillator, $V = \frac{1}{2}k(x^2+y^2)$, are of the form $\phi_{nm} = \phi_n(x)\phi_m(y)$ where $\phi_n(x)$ and $\phi_m(y)$ are ordinary one-dimensional harmonic oscillator wave functions

$$\phi_0(x) = \left(\frac{a}{\pi}\right)^{\frac{1}{4}} e^{-ax^2/2}, \quad \phi_1(x) = \left(\frac{a}{\pi}\right)^{\frac{1}{4}} \sqrt{2a}\, x\, e^{-ax^2/2}, \quad etc.$$

The energy of state ϕ_{nm} is $(n+m+1)\hbar\omega$. Find the first-order energy shifts and the zero-order energy eigenfunctions for the states $\phi_a = \phi_0(x)\phi_1(y)$ and $\phi_b = \phi_1(x)\phi_0(y)$ (which are initially degenerate) resulting from a perturbation of the form $H_1 = bxy$.