Physics 449 **HOMEWORK SET 3** **Spring 2009**

Due Wednesday February 11

12) A particle with spin 1 (three magnetic substates that are initially degenerate) is subjected to a spin dependent potential

$$V = AS_x^2 + BS_z^2.$$

Use degenerate-state perturbation theory to find the energy eigenvalues and the corresponding energy eigenvectors. The idea here is to think only about the spin part of the wave function and ignore the space part – equivalent to assuming the particle is at rest. The spin operators for $s=1$ are

$$S_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad S_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

13) (a) Use the perturbation theory result for the electronic energy, $\Delta E_{ee} = \frac{5}{8}\frac{Ze^2}{4\pi\epsilon_0}\frac{1}{a_0}$, to predict the energy required to remove one electron from neutral He, Li^+, Be^{++} and B^{+++}, (each of which has 2 electrons). [Hint: The energy to remove one electron is the same as the energy difference between the two-electron system (which is what we are presently calculating) and the one-electron system (which we know from the hydrogen-atom calculations).]

(b) Compare your results with the experimentally measured ionization energies, 24.6 eV for He, 75.6 eV for Li^+, 153.9 eV for Be^{++} and 259.4 eV for B^{+++}.

14) Two non-interacting spin-$\frac{1}{2}$ fermions are placed in an infinite square well potential that extends from 0 to L. The single-particle wave functions for this well are $\phi_n = \sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x$.

(a) Write down the formula for the ground-state wave function of the two-particle system including the spin wave function. Remember that the wave function must be antisymmetric under interchange of the two particles.

(b) When one particle is in the $n = 1$ state and the other is in the $n = 2$ state we obtain four distinct states, one with s = 0 and three with s = 1. Write down the wave functions of these 4 states.

(c) Let's now imagine that the two particles are attracted to each other by a weak linear force (proportional to the separation) which gives rise to a potential $v = \frac{1}{2}\mathrm{k}(\mathrm{x}_1 - \mathrm{x}_2)^2$. Assuming that this new potential can be treated in first-order perturbation theory, find the resulting energy difference between the S = 0 and S = 1 states of part (b) above. [Hints: Since you want the energy difference you only need to calculate the exchange energy. If you write v in the form $v = \frac{1}{2}\mathrm{kx}_1^2 + \frac{1}{2}\mathrm{kx}_2^2 - \mathrm{kx}_1\mathrm{x}_2$, the x_1 and x_2 integrals separate and several of the terms can be discarded immediately by noting that the integrands are odd about the center of the well.]