Physics 449 **HOMEWORK SET 8** **Spring 2009**

Due Wednesday April 8

31) Calculate the lifetime of the 3s state in hydrogen. Start by using the electric dipole selection rules to decide which states the 3s state can decay to. When there is more than one possible final state, the total decay probability per unit time (λ) is the sum of the λ's for the individual transitions.

32) In this problem we will derive a new relationship between the rates for absorption and emission of photons. Imagine a closed box of volume V which contains electromagnetic standing waves. As we showed in Physics 448, the number of standing wave modes per unit volume in the frequency interval $d\nu$ is given by

$$N(\nu)d\nu = \frac{8\pi}{c^3}\nu^2 d\nu.$$

The quantity $N(\nu)$ in this formula is known as Jeans number.

(a) Using this result, write an expression for $\rho(\omega)$ (where $\rho(\omega)d\omega$ is the **energy** per unit volume in the interval $d\omega$) in terms of $\overline{n}$, where $\overline{n}$ is the average number of photons in each mode. To obtain the result you need to change variables from ν to $\omega = 2\pi\nu$, and remember that the energy of each photon is $\hbar\omega$.

(b) Using this $\rho(\omega)$ write our formulas for the absorption rate, $A\rho(\omega)$, and the emission rate, $A\rho(\omega)+B$, in terms of $\overline{n}$. The formulas for A and B were derived in class. You should find that the absorption rate is $\overline{n}$ times constants while the emission rate $\overline{n}+1$ times the same constants.

33) Consider a system of equally spaced energy levels, $\epsilon_\mathrm{s} = (s+\frac{1}{2})\epsilon_0$ where $s = 0, 1, 2, \cdots$ with degeneracy $g_s = 2$ for each level.

(a) Find the total energy of the system at $T=0$ (i.e. the energy of the ground state) if the system contains 12 identical fermions.

(b) Suppose we now "raise the temperature" by adding $4\epsilon_0$ of energy. Make a table listing all possible energy distributions and indicate the number of distinct arrangements for each case. [Hint: I find 9 different distributions and a total of 42 distinct arrangements.]

(c) Find the average population $\langle n_\mathrm{s}\rangle$ of each level.

(d) Make a plot showing $\langle n_\mathrm{s}\rangle$ as a function of energy. Estimate the "temperature" from the graph and give T in units of ϵ_0/k. Then plot on the same graph the distribution predicted by the Fermi-Dirac formula for your value of T. [Hint: To find T first choose μ so that $n_\mathrm{s}/g_\mathrm{s}$ is $\frac{1}{2}$ at the right energy. Then make a guess for T by calculating the expected value of $n_\mathrm{s}/g_\mathrm{s}$ at $\epsilon = \mu + kT$ and $\epsilon = \mu - kT$.]

34) (a) Find the Fermi energy of copper. The density of Cu is $8.92\,\mathrm{g/cm^3}$, and there is one conduction electron per atom.

(b) Estimate the number of conduction electrons per unit volume with energy greater than $\epsilon_\mathrm{F}+ 0.1\,\mathrm{eV}$ for copper at room temperature ($kT = 0.025\,\mathrm{eV}$).

(c) Estimate the electronic specific heat at room temperature and compare with the atomic specific heat $C_v = 3R/mole$.

35) Find the energy difference between ϵ_F and the chemical potential μ for a system with $kT = 0.01\,\epsilon_\mathrm{F}$ and $g(\epsilon) = C\epsilon^{1/2}$. The starting point is to recognize that the total number of particles is the same at

all temperatures. At $T = 0$ all states are filled up to ϵ_{F}, so

$$\int_0^\infty g(\epsilon) f(\epsilon) d\epsilon = \int_0^{\epsilon_F} g(\epsilon) d\epsilon \qquad \text{where} \qquad f(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/kT}+1}.$$

To begin divide the left-hand integral into one that extends from 0 to ϵ_{F} plus a second that runs from ϵ_{F} to ∞. Then combine the two $(0,\epsilon_{\mathrm{F}})$ integrals and combine the terms using the identity

$$\frac{1}{e^x+1} + \frac{1}{e^{-x}+1} = 1.$$

Next change the integration variables using $x = \frac{(\epsilon-\mu)}{kT}$ in the $(\epsilon_{\mathrm{F}},\infty)$ integral. For the $(0,\epsilon_{\mathrm{F}})$ integral use $x = -\frac{(\epsilon-\mu)}{kT}$ and interchange the integration limits to absorb the $-$ sign. All the way through you need to be careful about the integration limits. Now assume that $\mu >>$ kT, which lets you extend one of the integration limits to ∞. At this point notice that the integrands go quickly to zero for $x >> 1$ and so we only need $g(\epsilon)$ for ϵ close to μ. So, make a Taylor series expansion of g about ϵ_F or μ and try to evaluate the integrals. It is safe to use the approximations $kT << \epsilon_F$ and $\epsilon_F - \mu << kT$. You will probably need $\int_0^\infty \frac{x}{e^x+1} dx = \frac{\pi^2}{12}$.