**Physics 449** **HOMEWORK SET 9** **Spring 2009**

**Due Friday April 17**

36) In this problem we will estimate the energy gap at $k = \pi/a$ for a periodic potential

$$V(x) = V_0 \cos \frac{2\pi}{a} x.$$

Assume the wave function is of the form $\psi(x) = u(x)e^{ikx}$, and take

$$u(x) = A(1 + be^{-2\pi ix/a}),$$

which is vaguely like what we would get if we take an arbitrary periodic function, $u(x)$, Fourier analyze it, and keep only the first terms. Substitute the proposed wave function into the Schrodinger equation. Try to identify the lowest frequency terms – ones that go like $e^{\pm i\pi x/a}$ as $k \to \pi/a$ – and discard the higher frequency (e.g. $e^{\pm 3i\pi x/a}$) terms. You should then be able to derive the following equations that relate $E$, $k$ and the parameter $b$:

$$\frac{\hbar^2 k^2}{2m} + \frac{V_0}{2} b = E \qquad \text{and} \qquad \frac{\hbar^2 (k - 2\pi/a)^2}{2m} b + \frac{V_0}{2} = Eb$$

(a) Solve for $E$ and $b$ just above and just below $k = \pi/a$. [Write $k = \pi/a \pm \delta$].

(b) Find the discontinuity is $E$ as $k = \pi/a$.

37) (a) Use the formula

$$\cos ka = \cos \alpha a - P \frac{\sin \alpha a}{\alpha a}$$

to determine the width of the first energy gap, $\Delta E$, for $P = 0.2$, $0.5$, and $0.8$. You should be able to express the answer in the form $\Delta E = C\pi^2\hbar^2/2ma^2$ where $C$ is some dimensionless number.

(b) Compare youre results from (a) with predictions from the approximation formula

$$\Delta E = \frac{2}{a} \left| \int_0^a V(x)\, e^{2\pi ix/a} dx \right| .$$

This equation comes from Fourier analyzing $V(x)$ and retaining only the leading term to get a potential like the one in Problem 36. Remember that the formula in (a) corresponds to a square well with $b \to 0$ and $V_0 \to \infty$ in such a way that $V_0 b$ is finite.

38) Estimate the number of electrons/cm$^3$ in the conduction band for Si and Ge at $T = 273\,\text{K}$. The energy gaps are $1.14\,\text{eV}$ in Si and $0.68\,\text{eV}$ in Ge. The densities are $2.33\,\text{g/cm}^3$ and $5.32\,\text{g/cm}^3$, and the atomic weights are 28.1 and 72.6 for Si and Ge respectively. Take $\mu$ to be at the midpoint of the energy gap. To estimate $g(\epsilon)$ at the bottom of the conduction band use the free electron result with $\epsilon \simeq 8\,\text{eV}$.

39) If liquid helium is treated as an ideal (noninteracting) Bose-Einstein gas, approximately what fraction of the atoms would be in the ground state at $T = 1\,\text{K}$? Take the density to be $0.125\,\text{g/cm}^3$.