

2004 UW Integration Bee

Preliminary Competition

January 27, 2004

Welcome to the first-ever UW Integration Bee! Only a writing implement will be necessary. Specifically, no textbooks, notes, tables of integrals, calculators, math T-shirts, tattoos, or copies of Teixeira's "Winning Integration Competitions Made Easy" are allowed. You will have 60 minutes to work on the 50 integrals in the exam. Answers must be circled. No partial credit will be given. All problems count equally. You may not start writing until told to begin, and you must stop writing when time is called. Answers will be posted by Thursday, 1/29 outside 1500 Sterling. The top 10 scores advance to the finals. If you are one of the top 10, you will be notified by phone or e-mail by Friday evening.

Note: $\ln x$ denotes the natural logarithm, to the base e ($=2.71828\dots$)

Name:

e-mail:

Phone:

undergraduate/graduate (circle one)

1. $\int x^{2004} dx$

$\frac{x^{2005}}{2005} + c$

2. $\int \sec x \tan x \, dx$

$$\sec x + c$$

3. $\int \frac{dx}{x^2 - 4}$

$$\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + c$$

4. $\int \frac{\sin x}{3 + 4 \cos x} \, dx$

$$-\frac{1}{4} \ln |3 + 4 \cos x| + c$$

5. $\int e^{\sin^2 x} e^{\cos^2 x} \, dx$

$$e^x + c$$

6. $\int \frac{dx}{x \ln x}$

$$\ln |\ln x| + c$$

7. $\int \frac{dx}{x^2 - 2x + 5}$

$$\frac{1}{2} \tan^{-1} \left(\frac{x-1}{2} \right) + c$$

8. $\int x^3 e^{x^2} dx$

$$\frac{1}{2}(x^2 - 1)e^{x^2} + c$$

9. $\int \sin x \sin 2x dx$

$$\frac{2}{3} \sin^3 x + c$$

10. $\int \sqrt{\frac{1+x}{1-x}} dx$

$$\sin^{-1} x - \sqrt{1-x^2} + c$$

11. $\int \frac{dx}{e^x + 1}$

$$x - \ln|e^x + 1| + c$$

12. $\int (\sinh x - \cosh x) dx$

$$e^{-x} + c$$

$$13. \int \frac{dx}{e^x + e^{-x}}$$

$$\tan^{-1} e^x + c$$

$$14. \int e^{x^x} (\ln x + 1) x^{2x} dx$$

$$(x^x - 1) e^{x^x} + c$$

$$15. \int \sin^{-1} x dx$$

$$x \sin^{-1} x + \sqrt{1-x^2} + c$$

$$16. \int \sqrt{\tan x} dx$$

$$\frac{1}{2\sqrt{2}} \left[\ln \left| \frac{1 - \sqrt{2 \tan x} + \tan x}{1 + \sqrt{2 \tan x} + \tan x} \right| + 2 \tan^{-1} \left(\frac{\sqrt{2 \tan x}}{1 - \tan x} \right) \right] + c$$

$$17. \int \sec^2 x dx$$

$$\tan x + c$$

$$18. \int \sec^3 x dx$$

$$\frac{1}{2} (\ln |\sec x + \tan x| + \tan x \sec x) + c$$

$$19. \int \frac{2x - 9\sqrt{x} + 9}{(x - 3\sqrt{x})^{1/3}} dx$$

$$\frac{6}{5} (x - 3\sqrt{x})^{5/3} + c$$

$$20. \int \pi^x dx$$

$$\frac{\pi^x}{\ln \pi} + c$$

$$21. \int \frac{dx}{x^2 + 4x + 4}$$

$$\frac{-1}{x+2} + c$$

$$22. \int \frac{dx}{3x^2}$$

$$\frac{-1}{3x} + c$$

$$23. \int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$$

$$2 \ln |1 + \sqrt{x}| + c$$

$$24. \int \frac{e^{1/x}}{x^3} dx$$

$$e^{1/x} \left(1 - \frac{1}{x}\right) + c$$

25. $\int \cos(\sin x) \cos x \, dx$

$$\sin(\sin x) + c$$

26. $\int \frac{dx}{x \ln x^5}$

$$\frac{1}{5} \ln|\ln x| + c$$

27. $\int \frac{x^2 + 3}{x^2 - 3} \, dy$

$$\frac{x^2 + 3}{x^2 - 3} y + c$$

28. $\int \cos \sqrt{x} \, dx$

$$2(\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}) + c$$

29. $\int e^{(x^2 - 3x + 2)^{-1}} \, dx$

$$c \frac{|x-2|}{|x-1|}$$

30. $\int \frac{e^{5x} + e^{7x}}{e^x + e^{-x}} \, dx$

$$\frac{1}{6} e^{6x} + c$$

$$31. \int x^{20} \ln x \, dx$$

$$\frac{x^{21}}{21} \left(\ln x - \frac{1}{21} \right) + c$$

$$32. \int (\tan^3 x + \tan^5 x) \, dx$$

$$\frac{\tan^4 x}{4} + c$$

$$33. \int \frac{x^5}{\sqrt{1+x^2}} \, dx$$

$$\frac{1}{5}(1+x^2)^{5/2} - \frac{2}{3}(1+x^2)^{3/2} + (1+x^2)^{1/2} + c$$

$$34. \int \frac{|\sin x + \cos x|}{\sqrt{1 + \sin 2x}} \, dx$$

$$x + c$$

$$35. \int \frac{dx}{2 + \cos x}$$

$$\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$$

$$36. \int e^x \tan e^x \, dx$$

$$\ln |\sec e^x| + c$$

$$37. \int \frac{x}{\sqrt{1-x^2}} dx$$

$$-\sqrt{1-x^2} + c$$

$$38. \int \frac{x^3 + 3x^2 + 3x}{x^4 + 4x^3 + 6x^2 + 4x + 1} dx$$

$$\ln|x+1| + \frac{1}{3(x+1)^3} + c$$

$$39. \int \frac{dx}{\sqrt{1+\sqrt{x}}}$$

$$\frac{4}{3}(1+\sqrt{x})^{3/2} - 4(1+\sqrt{x})^{1/2} + c$$

$$40. \int \frac{x}{x^4+1} dx$$

$$\frac{1}{2} \tan^{-1}(x^2) + c$$

$$41. \int \cot x dx$$

$$\ln|\sin x| + c$$

42. $\int 42 \, dx$

$$42x + c$$

43. $\int \frac{dx}{1 - (n+1)\sin^2 3x}$

$$\frac{1}{6\sqrt{n}} \ln \left| \frac{1 + \sqrt{n} \tan 3x}{1 - \sqrt{n} \tan 3x} \right| + c$$

44. $\int \frac{\ln(\cos x)}{\cot x} \, dx$

$$-\frac{1}{2} (\ln |\cos x|)^2 + c$$

45. $\int 100\$ \, d\$$

$$50\$^2 + c$$

46. $\int \frac{x^2}{\sqrt{x}} \, dx$

$$\frac{2}{5} x^{5/2} + c$$

47. $\int \frac{\tan^{-1} x}{x^2} \, dx$

$$\frac{-\tan^{-1} x}{x} + \ln \left| \frac{x}{\sqrt{1+x^2}} \right| + c$$

48. $\int e^{3\ln x} dx$

$$\frac{x^4}{4} + c$$

49. $\int \sqrt{x^2 - 1} dx$

$$\frac{1}{2}x\sqrt{x^2 - 1} - \frac{1}{2}\ln|x + \sqrt{x^2 - 1}| + c$$

50. $\int \frac{dx}{\sec x + \tan x}$

$$\ln|1 + \sin x| + c$$