Overview of
The Physics of Running

Jim Reardon
UW Physics Department/Wisconsin Track Club
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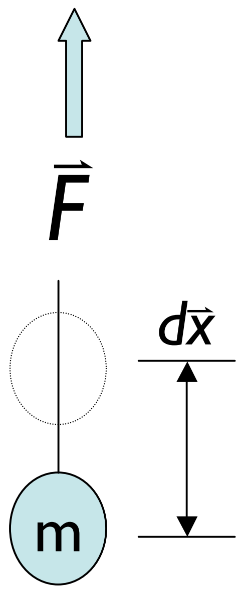
Overview of
The Physics of Running

1. Runners
2. Muscle characteristics
3. Fundamental parameters
4. Cross-species comparisons
5. Passive Dynamic Walking Robots
6. Conclusions

Runner #1: Paula Radcliffe

Runner #2: Haile Gebrselassie

Mechanical Work W



$$\text{work} = \text{force} \times \text{distance}$$

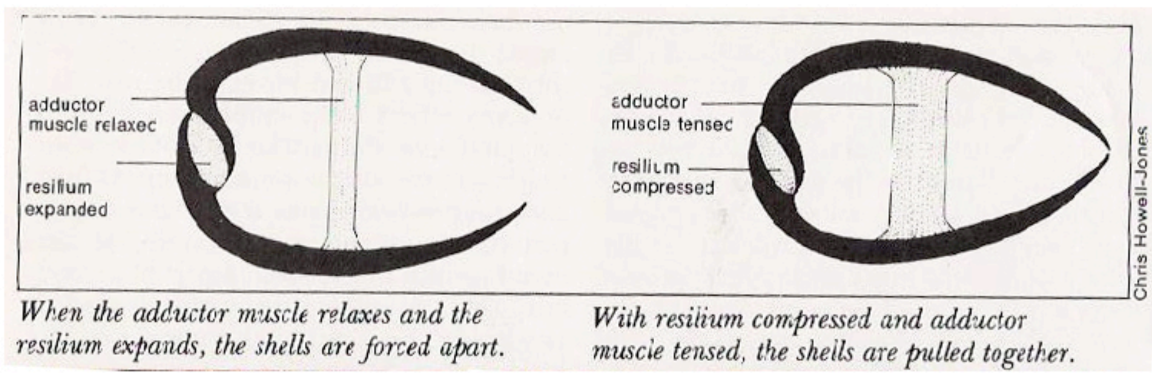
$$dW = \vec{F} \cdot d\vec{x}$$

$$W = \int \vec{F} \cdot d\vec{x}$$

Agrees with common-sense definition of "work"--
lifting things takes work
Disagrees with common-sense definition of "work"--
holding things in place doesn't take work

Vertebrate and Invertebrate Muscle

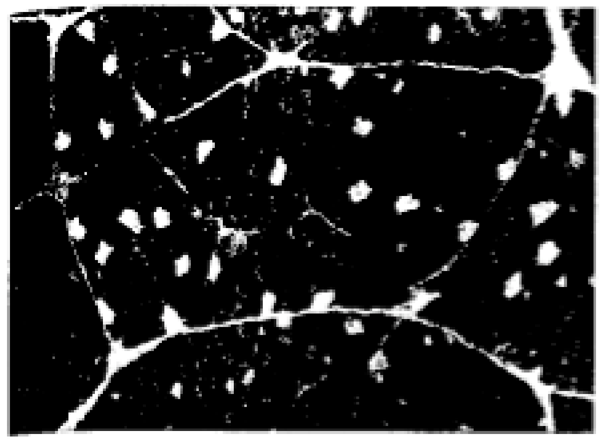
Clam Muscle: when tensed, "sets" in place. Tension can be maintained for hours. Slow.



From International Wildlife Encyclopedia, Vol 4, Marshall Cavendish, 1969

Vertebrate Muscle: Does not "set" when tensed. Fast.

Vertebrate Muscle



Cross-section of cat tibialis anterior muscle, showing muscle fibers belonging to the one motor unit (Roy et al,, Muscle And Nerve, 18, 1187 (1995))

Normal activity: motor units fire independently at 5 Hz--30Hz

Advantages: speed, redundancy, stiffness, stability
Disadvantages: fatigue, heat generation

Heat/Work Ratio in Runners is about 4

There does not exist a quantitative model relating muscle activity to use of chemical energy from food.

Heat/work estimated from measurements of gas exchange, temperature and cooling rate, force measurements…

How much heat is due to “friction” between motor unit fibers?

Chaos in support muscle?

Normal activity: motor units fire independently at 5 Hz--30Hz

Fatigued muscle: oscillations visible at <5Hz ("sewing machine leg"). Period doubling?

Elite Marathon Runners are Limited by Heat Dissipation

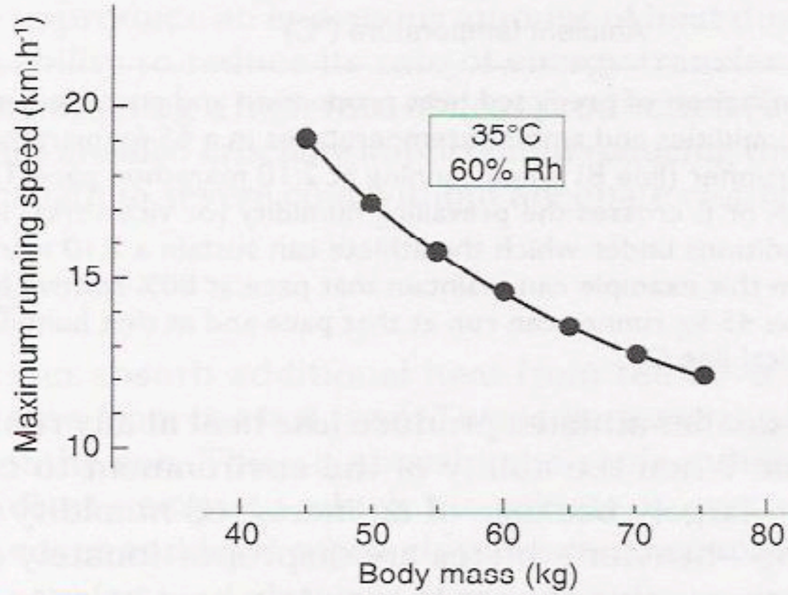


Figure 4.6 Predicted maximum running speeds that athletes of different masses can sustain in hot environmental conditions (35°C; 60% relative humidity [Rh]).

From Noakes, Lore of Running 4th ed., 2003

Heat Dissipation Depends on Scale

Size of runner ~ L

Rate of heat generation ~ working muscle volume ~ L^3

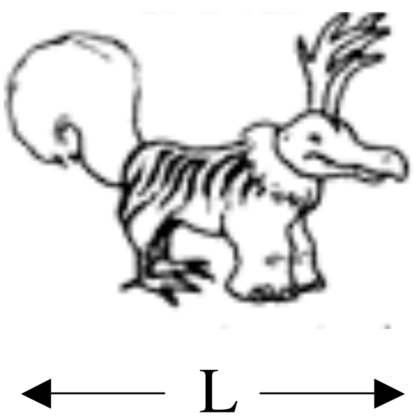
Rate of heat loss (evaporation of sweat, wind) ~ surface area~L^2

As body scale L increases, runners have more trouble staying cool.

Smaller runners do better in extreme heat than larger runners.

Athens 2004 Women's Olympic Marathon: 35 C

Galileo's Jumping Argument (1638)



- Size of Animal = **L**
- Cross-sectional area of muscle ~$\mathbf{L^2}$
- Force ~ cross-section
- Maximum change in length of muscle ~ **L**
- Work = force x distance ~ $\mathbf{L^3}$
- Weight ~ volume ~ $\mathbf{L^3}$
- Jump height= **c** x **(Work)/(Weight)**

--> Independent of Animal Size!

Human parameters from Physical Arguments

(Barrow & Tipler, c. 1985)

The fundamental constants of quantum electrodynamics, plus G:

$c = 3 \times 10^{8}$ m/s	the speed of light
$\hbar = 1.05 \times 10^{-34}$ Js "h – bar"	the quantum of angular momentum
$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^{2}\,kg^{-2}}$	"the gravtational constant"
$\alpha = e^{2}/4\pi e_{0}\hbar c = 1/137.04$	"the fine - structure constant"
$\beta = m_{e}/m_{p} = 1/1836.12$	ratio of electron to proton mass
$m_{e} = 9.109 \times 10^{-31}$ kg	mass of electron

Size of Atoms

Size of atoms set by Uncertainty Principle and Virial Theorem:

$$\Delta p \Delta x \geq \hbar \qquad K.E = -\frac{1}{2} P.E$$

Using

$$K.E. = p^2 / 2m_e\,, \;\; P.E. = -Ze^2 / 4\pi\varepsilon_0 r, \;\; r \sim \Delta x,$$

$$r_0 \sim \frac{\hbar}{c} \frac{1}{Z\alpha\, m_e} = 5 \times 10^{-11}\ m$$

Z is the nuclear charge on the atom.

Density of Matter

$$\rho_0 \sim \frac{m_p}{\left(2r_0\right)^3} = \frac{m_e^4}{8\beta}\frac{c^3\alpha^3}{\hbar^3} = 1.4\ \mathrm{g/cm^3}$$

compare $$\frac{\rho_{U-238}}{\rho_{H-1}} = \frac{18.95\ \mathrm{g/cm^3}}{0.076\ \mathrm{g/cm^3}} = 250$$

Binding Energies of Atoms

Binding Energy=-(P.E.+K.E.)=-1/2 P.E.

$$\text{Binding Energy} = \frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0 r_0} = \frac{1}{2} m_e Z^2 \alpha^2 c^2$$

$$= \ 13.6 \text{ eV} \times \text{Z}^2 \equiv 1\,Ry$$

Liquid--Gas Transition Temperature

Constant T and V: Minimize Helmholz Free Energy U-TS
U=energy, T=temperature, S=entropy

Can do this by making U large and negative (liquid) or by Making S large and positive (gas).

Expect liquid gas transition to happen at T where U~ TS

$$U/N \sim 1\,Ry,\ S/N = k_b(\log n_q / n + 5/2) \sim 15k_b$$

$$1\,Ry \sim 15k_b T_{lg} \rightarrow T_{lg} \sim 10{,}000\ K$$

True (no liquids persist for $T>T_{lg}$) but overestimates T_{life}…

Vibrational Energies of Molecules

Imagine atoms and molecules are held together by springs.

Frequency of oscillation $\omega \sim \dfrac{1}{\sqrt{m}}$

Energy associated with atomic vibrations ~ 1 Ry

Energy associated with molecular vibrations $E_m \sim \sqrt{\dfrac{m_e}{m_p}}\, Ry$

Ansatz: Life Exists Due To Interplay Between Molecular Binding Energies and Vibrations

$$k_b T_{life} \sim \sqrt{\frac{m_e}{m_p}}\ Ry \sim 250\ K$$

Note: this is very nearly an ad hoc argument

Gravity I

"Gravitational Fine Structure Constant":

$$\alpha_G \equiv \frac{Gm_p^2}{\hbar c} = \frac{G\beta^2 m_e^2}{\hbar c} \sim 5.9 \times 10^{-39}$$

Gravity is weaker than electromagnetism by a factor:

$$\frac{\alpha_G}{\alpha} = 8.1 \times 10^{-37}$$

Gravity II: Planets

Take N atoms of atomic number A and stick them together.
They will end up in a lump of size R.
The gravitational potential energy of the lump is:

$$P.E._G = -\frac{GM^2}{R} \sim -\frac{G(NAm_p)^2}{R}$$

Assume lump has atomic density ρ_0, so that $M \sim \frac{4\pi}{3}R^3\rho_0$.

Then escape velocity at surface is:

$$v_{esc} = \sqrt{\frac{8\pi}{3}GR^2\rho_0}$$

Life Needs Atmosphere

Life requires an atmosphere composed of something other than Hydrogen. Equate v_{esc} and thermal velocity of Hydrogen at T_{life}:

$$v_{esc} = \sqrt{\frac{8\pi}{3} G R^2 \rho_0} \approx \sqrt{\frac{k_b T_{life}}{m_p}}$$

Solving for the planetary radius R:

$$R_p \sim \sqrt{\frac{3 k_b T_{life}}{8\pi G \rho_0 m_p}} = 5 \times 10^3 \text{ km}$$

Compare R_{earth}=6.4x10^3 km.

Surface Gravity

The acceleration due to gravity at the surface of the planet can be calculated as:

$$a_g = \frac{GM}{R^2} = \frac{4\pi}{3} G R_p \rho_0 \sim 2 \text{ m/s}$$

This is smaller than the observed a_g~10 m/s. Using the observed average earth density ρ_{Earth}~5.5 g/cm^3 gives a_g~8 m/s

The Human Condition

Define human size L_h: we don't break if we fall down

Energy lost by falling down:

$$E_{fd} = \frac{m_h a_g L_h}{2} = a_g \rho_0 L_H^4$$

Energy required to cause excessive molecular vibration along fracture site:

$$E_{frac} = \frac{k_b T_{life}}{N_m}$$

Where N_m is the number of molecules bordering the fracture:

$$N_m = \left(\frac{m_H}{m_p}\right)^{2/3} = \left(\frac{\rho_0 L_H^3}{m_p}\right)^{2/3}$$

Deduce L_H~1.5 cm. Life is rugged!

The end of physics…

This could represent the size limit for land-dwelling creatures without skeletons (ie parts that don't boil around 250 C).

However, other authors (Press, 1983) have deduced a scale size of a few cm using different arguments.

Conclusions: Suspect any land-dwelling life we encounter will have evolved under a gravity similar to ours. Perhaps such life will be our size (or smaller).

To go further, need detailed knowledge of living beings.

Energy Cost of Locomotion ~ $m^{-1/3}$

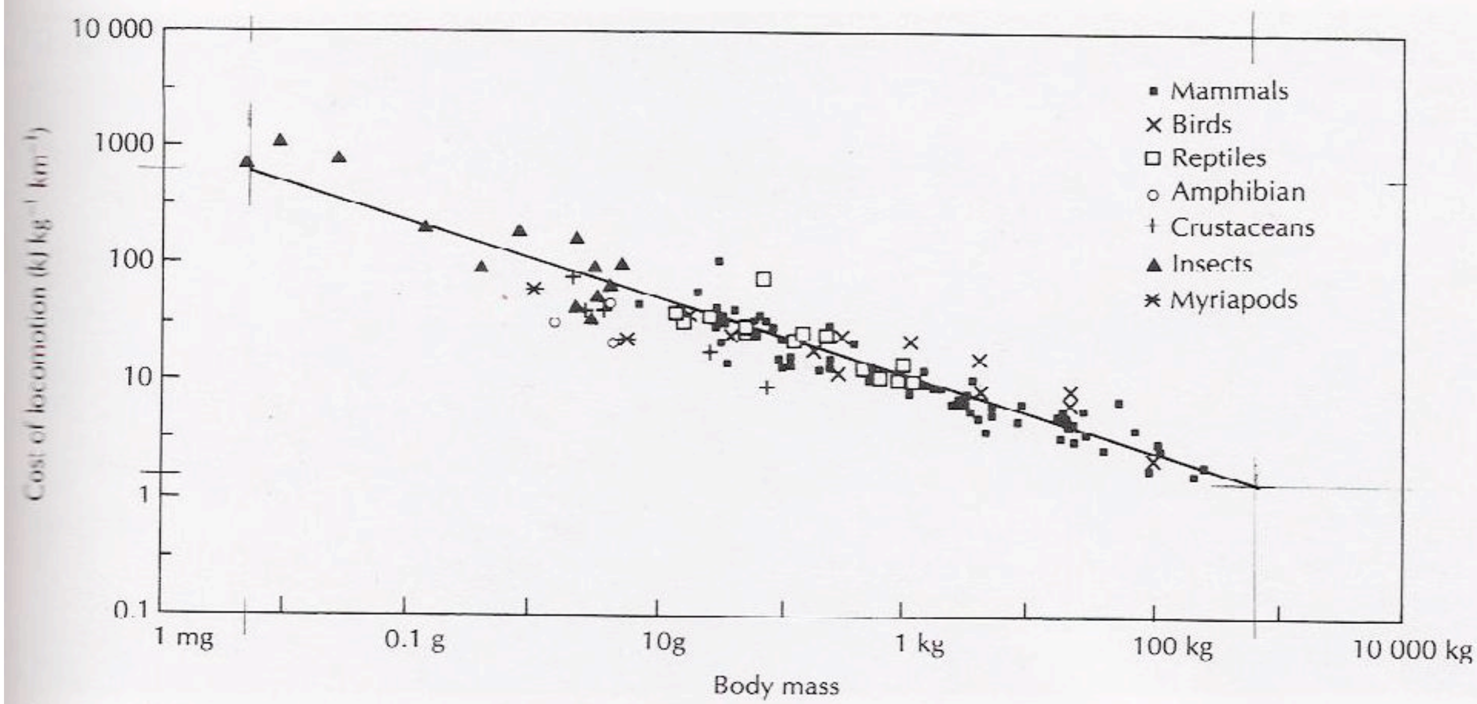


Figure 5.20 The cost of transport for animals of various types. To move a given distance small animals, regardless of type, consume more oxygen per unit mass than larger animals. This relationship holds over a wide range of body sizes and leg numbers, from elephants to centipedes. [Full and Tu 1991]

(data from Full and Tu, graph scanned from Animal Physiology, K. Schmidt-Nielsen, 5th ed., 1997)

Universal Cross-species Energy Cost is 0.75 J/kg/L

Define animal body length: $L = \left(\frac{3m}{4\pi\rho} \right)^{1/3} \propto m^{1/3}$

Then the data of Full and Tu predict that the energy cost of moving one body length is a constant across species:

$$E_L \approx 0.75 \;\; J/kg$$

Note also that different species move at different speeds…

Energy Cost of Running in Humans

$$E_{km} \approx 1 \;\; kcal/kg/km$$

You burn the same number of calories per kilometer, no matter how fast you run (about 4/3 the number predicted by the data of Full and Tu).

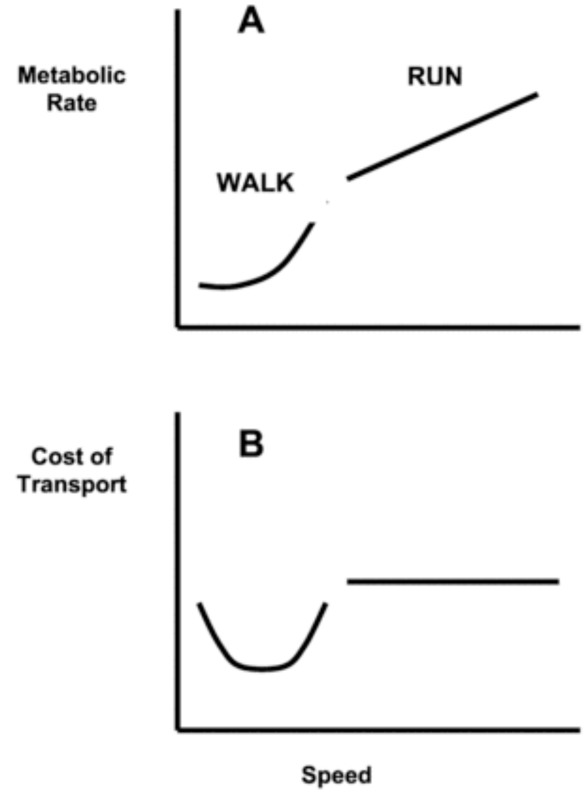
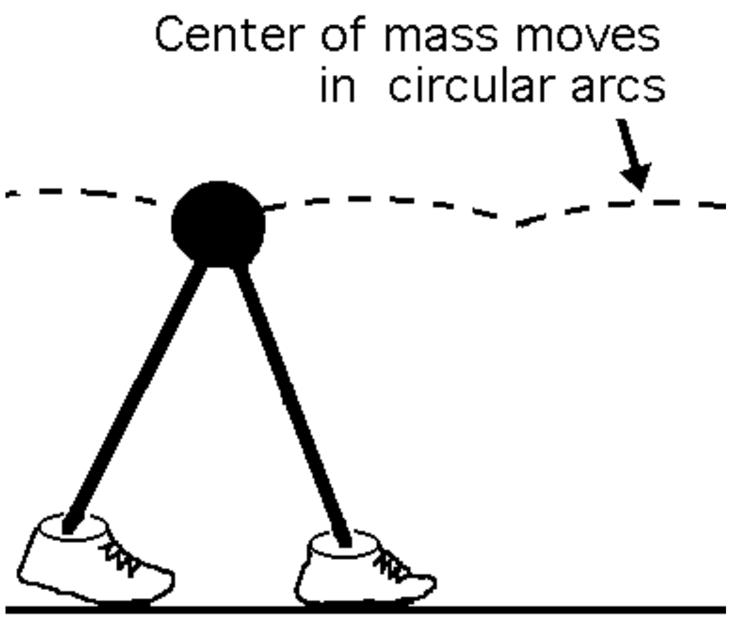


FIGURE D-18 The rate at which metabolic energy is transformed in relation to speed during steady-state human walking and running (A). Dividing metabolic rate by speed provides the energy expended per unit distance (B).
SOURCE: Reprinted, with permission Margaria et al. *JAP* (1963).

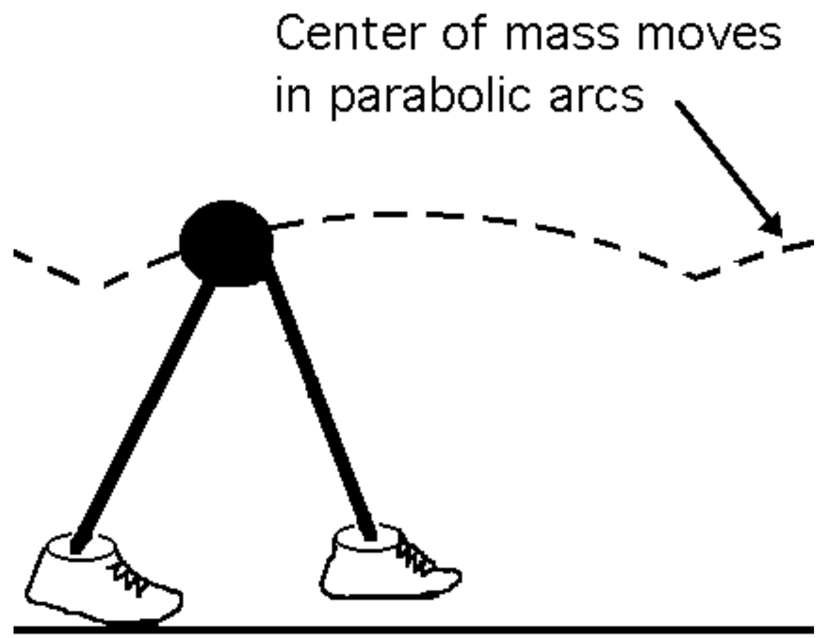
The Difference Between Running and Walking

Walking



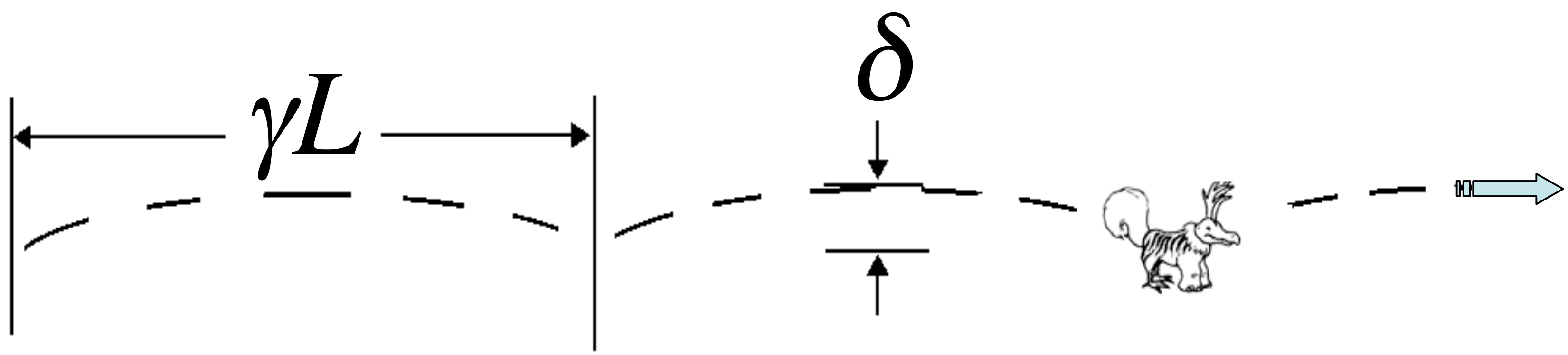
Center of mass is highest
when directly over support leg

Running



Center of mass is lowest
when directly over support leg

Energy Cost of Running Ansatz



Change in height of center of gravity during one stride: δ

Gravitational potential energy associated with this: $mg\delta$

Stride length: γL

Energy cost of running one body length: $\dfrac{mg\delta}{\gamma}$

Constraint on Stride Dimensions

It's as if $$E_L \approx 0.75\,J/kg \sim \frac{g\delta}{\gamma}$$

$$\frac{\delta}{\gamma} \sim 7.5\ cm$$

You have to go 7.5 cm in the air if you want to cover your body length in one step (ie small animals appear to 'hop').

Summary of Cross-Species Comparison

It is possible to understand the cross-species comparison data of Full and Tu as well as the strictly human data from Margaria by the ansatz that land-dwelling animals move horizontally for free but expend energy to move their centers of mass up and down.

There may be other explanations, and this one doesn't explain why small animals ought to have to hop.

A Do-it-yourself Passive Dynamic Robot

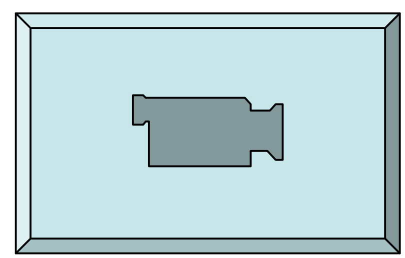
http://ruina.tam.cornell.edu/research/topics/locomotion_and_robotics/papers/tinkertoy_walker/tinkertoy_walker.mpg

Passive-Dynamic Walking Robots

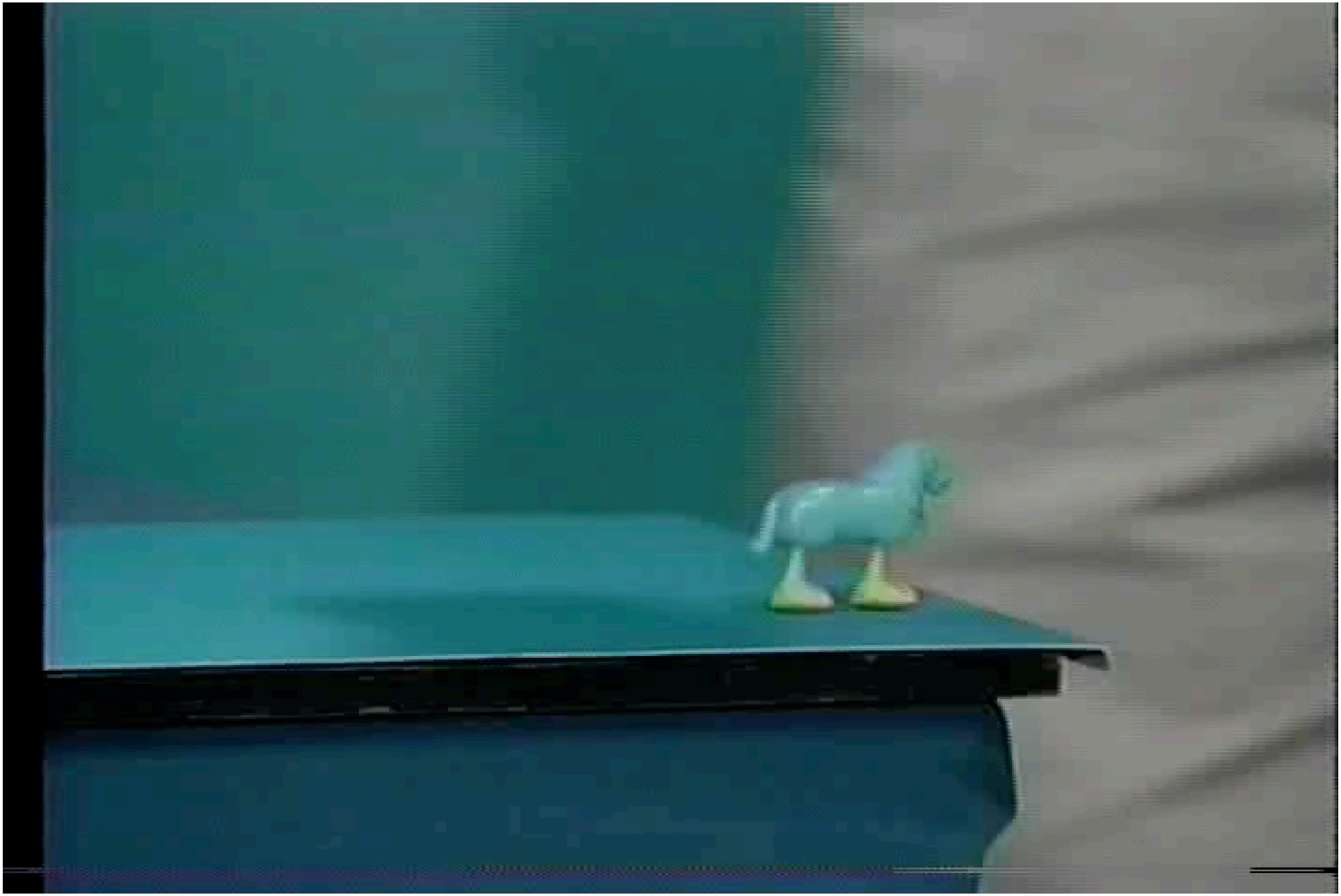
T. McGeer, 1990

"Passive Dynamic": no motors, no control system, no feedback loops

All motion a consequence of Newton's laws, including the effects of gravity and inertia (can be well-predicted by numerical integration of simple equations)



http://ruina.tam.cornell.edu/research/topics/locomotion_and_robotics/history/mcgeer_first_walker_with_knees.mov



Dynamic Stability Need Not Require a Stable Equilibrium

Equilibrium: $\frac{\mathbf{d}}{\mathbf{dt}} = \mathbf{0}$

Stability: small perturbations ε die away

$$\varepsilon(t \rightarrow \infty) = 0$$

Dynamic Stability: $F(t + \tau) = F(t)$

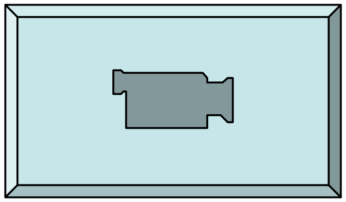
(for all F, for some τ)

Anthropoid Passive-Dynamic Walking Robots

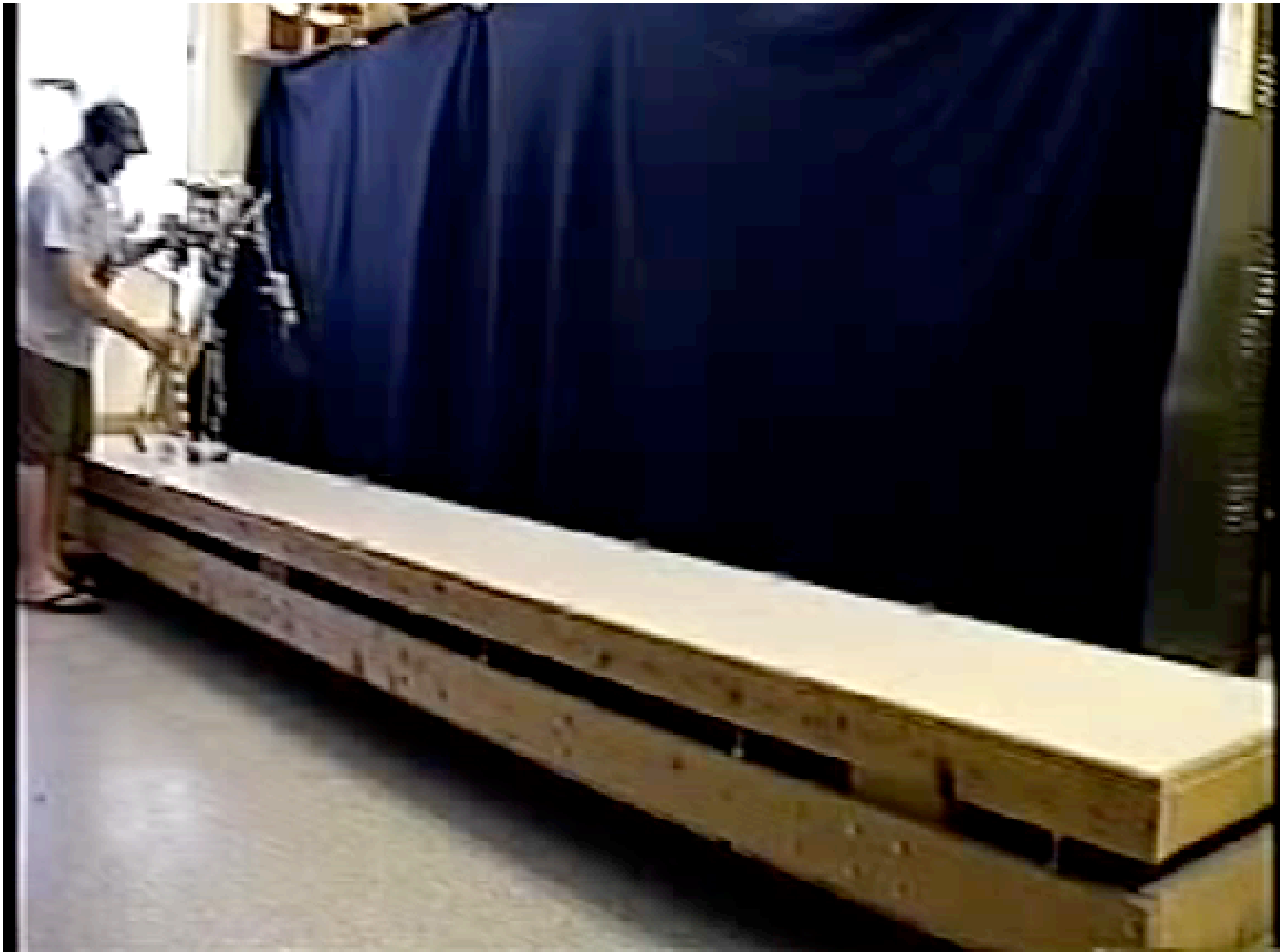
S. Collins et al., 2001

Energy source: gravitational potential energy
(walking down slope)

Major energy loss: heel strike



http://ruina.tam.cornell.edu/research/topics/locomotion_and_robotics/papers/3d_passive_dynamic/from_angle.mpg



Cornell Human Power Lab Autonomous Biped. Walks on level ground using 11 watts total. Powered by toe-off triggered by heel-strike. Steve Collins (&Andy Ruina) July-Aug 2003.

Wisdom from Robots

Heel strike is energetically costly.
Limb swinging is energetically cheap.

Models tend to show decreased cost of transport with decreased stride length and increased stride frequency (beyond the point of reason).

Practical Insights

1. Minimize up and down motion

   Humans can do this by shorter stride/higher cadence than the average runner exhibits.

2. Minimize energy loss at heel strike. Run quietly!

Practical Questions

1. The mysterious factor of 4. There is no explanation for why humans (and perhaps vertebrates) exhibit a heat/work ratio of 4. Is this trainable? (Small change in this number 4 would cause large change in race times).
2. Does high mileage lead to faster race times by decreasing “bobble” (aka $\delta/\alpha L$ ratio)?
3. Successful runners without exception have cadences of 180 steps/minute or above during races. To what extent is optimal cadence trainable?

Impractical Questions

1. Why do no multicellular living creatures feature rotary locomotion? (eg birds that look like helicopters, fish with propellors, or land-dwellling creatures on wheels).
2. Does there exist a gradient for which the optimal bipedal descent strategy is a skip rather than a run?
3. Would we expect to be able to compete against alien life in a 100 m dash, or marathon?