PHY 322 Fall 2004 Midterm I Solns 10/6/04

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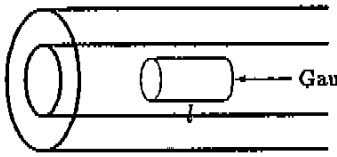
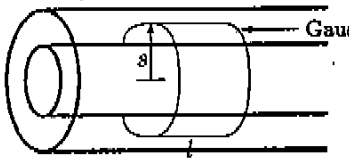
(i) 

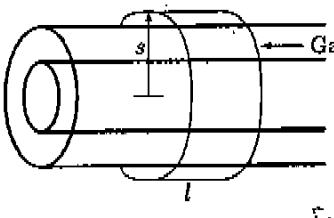
$$\oint \mathbf{E} \cdot d\mathbf{a} = E \cdot 2\pi s \cdot l = \tfrac{1}{\epsilon_0} Q_{\text{enc}} = \tfrac{1}{\epsilon_0} \rho \pi s^2 l;$$

$$\boxed{\mathbf{E} = \frac{\rho s}{2\epsilon_0} \hat{\mathbf{s}}.}$$

(ii) 

$$\oint \mathbf{E} \cdot d\mathbf{a} = E \cdot 2\pi s \cdot l = \tfrac{1}{\epsilon_0} Q_{\text{enc}} = \tfrac{1}{\epsilon_0} \rho \pi a^2 l;$$

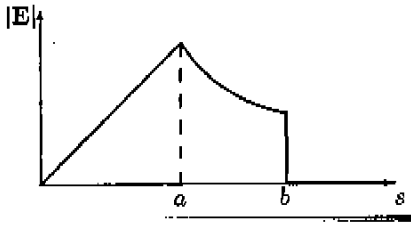
$$\boxed{\mathbf{E} = \frac{\rho a^2}{2\epsilon_0 s} \hat{\mathbf{s}}.}$$

(iii) 

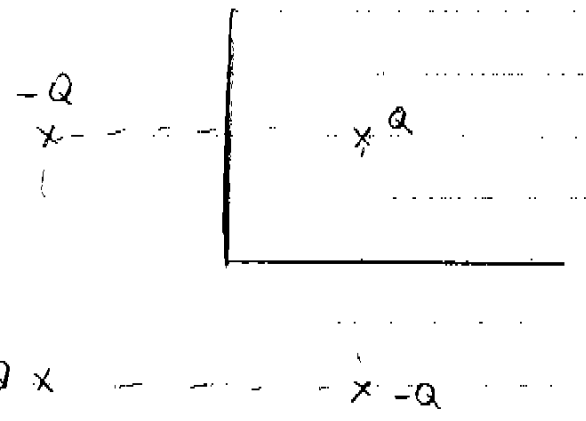
$$\oint \mathbf{E} \cdot d\mathbf{a} = E \cdot 2\pi s \cdot l = \tfrac{1}{\epsilon_0} Q_{\text{enc}} = 0;$$

$$\boxed{\mathbf{E} = 0.}$$

iv)



2. By the method of Image:



(a) The potential between the plates:

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{|\vec{r}-\vec{A}|} - \frac{1}{|\vec{r}-\vec{B}|} - \frac{1}{|\vec{r}-\vec{C}|} + \frac{1}{|\vec{r}-\vec{D}|}\right]$$

where $\vec{A} = (a, a)$

$\vec{B} = (-a, a)$

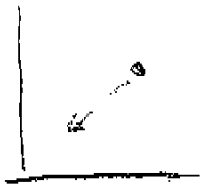
$\vec{C} = (a, -a)$

$\vec{D} = (-a, -a)$

(b) The force on charge Q is

$$\vec{F} = \frac{Q^2}{4\pi\epsilon_0}\left\{\frac{-\hat{x}}{(2a)^2} - \frac{\hat{y}}{(2a)^2} + \frac{\frac{1}{\sqrt{2}}(\hat{x}+\hat{y})}{(2\sqrt{2}a)^2}\right\}$$

$$= \frac{Q^2}{4\pi\epsilon_0}\frac{1}{a^2}\left(\frac{1}{4} - \frac{1}{8\sqrt{2}}\right)(-\hat{x}-\hat{y})$$



(c) To calculate the work needed, we can proceed in 2 ways:

$$W = \frac{1}{2}\sum_i q_i V(\vec{r}_i)$$

$$= \frac{1}{2}Q\left\{\frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{2a} - \frac{1}{2a} + \frac{1}{2\sqrt{2}a}\right)\right\}$$

$$= -\frac{Q^2}{4\pi\epsilon_0 a}\left(\frac{1}{2} - \frac{1}{4\sqrt{2}}\right)$$

[Note that no work is needed to bring the image charges in]

Alternatively

$$W = \int \vec{F} \cdot d\vec{x}$$

$$= \int_{\infty}^{a} \frac{Q^2}{4\pi\epsilon_0 x^2} \left(\frac{1}{4} - \frac{1}{8\sqrt{2}} \right) dx$$

$$+ \int_{\infty}^{a} \frac{Q^2}{4\pi\epsilon_0 y^2} \left(\frac{1}{4} - \frac{1}{8\sqrt{2}} \right) dy$$

$$= - \frac{Q^2}{4\pi\epsilon_0 a} \left(\frac{1}{2} - \frac{1}{4\sqrt{2}} \right) \quad \#$$

In agreement with above.

Comment: Work done is negative since charge Q is attracted to the plates.