Lecture 1: Conservation laws (3 Sep 14)

A. Administrative details

1. Syllabus has office hours and e-mail address; reference materials on Reserve in the Physics Library (4220 Chamberlin Hall); weekly problem assignments and daily text reading material. Lecture notes are on website.

2. Supplementary material to Fetter & Walecka is mainly from Goldstein; some cross-references to Landau-Lifshitz text (generally terser, more bare bones). Relativity here as add-on to the FW text.

3. Homework:
   (a) This is a ‘skills course’. Physics is not a spectator sport!
   (b) The homework is an essential part of the course. If you do not have enough time to work through the algebra, you should at least write down how you would set up the problem—what the problem is testing, what information is needed, and an outline of how the calculation would go.
   (c) The problems generally require several steps and it will take some practice to get a sense of ‘what works’ as far as setting up the problem.
   (d) Especially in the first problem sets, there will be parts that are done with “elementary considerations” and parts that require the full analytical theory.
   (e) On Fridays, we will talk about ingredients for next problem set; problems due Monday will be returned on Wednesday.

4. Background: anticipate that almost all of the students will have taken an upper-division undergraduate course in mechanics. The beginning of this semester will repeat/elaborate on some of those topics. The goal for this part will be: consolidate the previous work (Kepler problem, rotating frames, oscillators) and use some of the examples to introduce a higher level of applied math (matrix methods, calculus of variations).
5. The main parts of the course have reformulations of Newtonian mechanics that make some problems more tractable. Then there will be two topics at the boundary: special relativity (new physics); chaos (implicit in classical mechanics, but effectively a limit on “determinism,” even in classical mechanics). Many of the topics are chosen because they arise in the transition to quantum mechanics.

B. Newton’s laws – FW 1

1. This is the essence of classical mechanics – based on the intuitive concept of an inertial coordinate system. FW pg 1: “at rest with respect to the fixed stars” [old jargon, by now “which stars are they?” and more careful thought (Einstein) leads to general relativity]. B & O: pg.1: inertial frame is one in which Newton’s laws hold. Another version LL Sec 3: “frame of reference in which space is homogeneous and isotropic and time is homogeneous”

2. Typography: will generally use bold face for a vector: \( \mathbf{p} \equiv \vec{p} \).

3. N#1 In an inertial frame, every body remains at rest or in uniform motion unless acted on by a force \( \mathbf{F} \). \( \mathbf{F} = 0 \Rightarrow \mathbf{v} = (\text{constant}) \); more carefully: \( \mathbf{p} = (\text{constant}), \) velocity \( \mathbf{v} \), (mechanical) momentum \( \mathbf{p} = m\mathbf{v} \).

4. N#2 In the inertial frame, application of a force causes a change of momentum:
   \[
   \mathbf{p} = \frac{d\mathbf{p}}{dt} = \mathbf{F}
   \]
   and with constant mass \( m \),
   \[
   m\frac{d\mathbf{v}}{dt} = \mathbf{F}
   \]

5. N#3 To each action, there is an equal and opposite reaction. Force of 2 on 1, \( \mathbf{F}_{21} = -\mathbf{F}_{12} \), the force of 1 on 2. The forces act along the line separating 1 and 2 [restricted to central forces – more general situations arise in electromagnetism].
6. Given these principles, the task in mechanics is to solve for the motion under given forces. Two important central force examples are the inverse square law forces (e.g., “The Unreasonable Effectiveness of Mathematics in the Natural Sciences” – Eugene Wigner): Newtonian gravitation and Coulomb forces

\[ F_{21} = -G m_1 m_2 \frac{r_1 - r_2}{|r_1 - r_2|^3} \]

\[ F_{21} = \frac{Q_1 Q_2}{4\pi\epsilon_0} \frac{r_1 - r_2}{|r_1 - r_2|^3} \]

Parallels between the Kepler problem of gravitation and Rutherford scattering.

7. Galilean invariance, a frame moving at constant velocity \( V \) relative to an inertial frame is itself an inertial frame:

\[ s = r + Vt \Rightarrow d^2r/dt^2 = d^2s/dt^2 \]

and \( F(r_i - r_j) = F(s_i - s_j) \) so forces and accelerations are the same in the two frames. [a special functional dependence: these statements have not been “fully hedged.”]

**C. Linear and angular momentum**

1. Conservation laws on “totals” for systems with pair-wise forces. These turn out to be related to symmetries under translations and rotations.

2. Linear momentum \( p \); for \( N \)-particles, \( j = 1,..N \), \( p_j \) total momentum

\[ P = \sum_{j=1}^{N} p_j \]

Then, if the only forces are “internal forces” satisfying Newton #3,

\[ \frac{dp_i}{dt} = \sum_{j \neq i} F_{ji} \]

\[ \frac{dP}{dt} = F_{tot} = \sum_i \sum_{j \neq i} F_{ji} = 0 \]

canceling pairs \( F_{ab} + F_{ba} = 0 \). Total momentum conserved.
3. Angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ and with $\mathbf{p} = m\mathbf{v}$, the time derivative is 
\[
\frac{d\mathbf{L}}{dt} = \mathbf{r} \times \frac{d\mathbf{p}}{dt} = \mathbf{r} \times \mathbf{F}
\]

4. Many particles:
\[
\mathbf{L} = \sum_j \mathbf{r}_j \times \mathbf{p}_j; \quad \frac{d\mathbf{L}}{dt} = \sum_j \mathbf{r}_j \times \mathbf{F}_j
\]

Remark I: if $\mathbf{F}_{\text{tot}} = 0$ the $d\mathbf{L}/dt$ is independent of the choice of origin for the $\mathbf{r}_j$. Remark II, for internal central forces,
\[
\frac{d\mathbf{L}}{dt} = \sum_i \mathbf{r}_i \times \sum_{j \neq i} \mathbf{F}_{ji} = \frac{1}{2} \sum_i \sum_{j \neq i} (\mathbf{r}_i - \mathbf{r}_j) \times \mathbf{F}_{ji} = 0
\]

with last equality because each term in summand vanishes under central forces. Conservation of total angular momentum.

5. Conservative forces: $\nabla \times \mathbf{F} = 0$ gives with Stokes law
\[
0 = \int (\nabla \times \mathbf{F}) \cdot d\mathbf{A} = \oint_C \mathbf{F} \cdot d\mathbf{\ell}
\]
Then the line integral of $\mathbf{F} \cdot d\mathbf{\ell}$ between two arbitrary points $a$ and $b$ is independent of path. Hence to a potential energy function $\Phi$, $\mathbf{F} \equiv -\nabla \Phi$,
\[
\int_a^b \mathbf{F} \cdot d\mathbf{\ell} = -[\Phi(b) - \Phi(a)]
\]
\[
\int d\mathbf{\ell} \cdot \frac{d\mathbf{v}}{dt} = \int dt \mathbf{v} \cdot \frac{d\mathbf{v}}{dt} = \frac{1}{2}[v_f^2 - v_i^2]
\]
and then to
\[
\frac{1}{2}mv_i^2 + \Phi(1) = \frac{1}{2}mv_f^2 + \Phi(2)
\]
(Cartesian coordinates, mechanical momentum ... will be greatly generalized.....)