1) Charge, \( Q \), on capacitor makes a voltage \( Q = C \Delta V \Rightarrow \Delta V = \frac{Q}{C} \)

2) That produces a current \( I = \frac{\Delta Q}{\Delta t} = \frac{Q}{RC} \)

3) Current flow \( (I = \frac{\Delta Q}{\Delta t}) \) means \( Q \) on capacitor is decreasing
\[
Q(t) = Q_0 e^{-\frac{t}{RC}} \quad \text{where} \quad T = RC
\]

Features of the exponential function

1) The time, \( t \), to for the charge to fall to half its previous value is always the same.

2) The slope at any point is \( \propto \) the value of the function.

3) To find the time required to reach a certain point use natural logs. For example, the time to fall to \( \frac{1}{2} Q_0 \) is as follows
\[
Q = Q_0 e^{-\frac{t}{RC}}
\]

Set \( Q = \frac{1}{2} Q_0 \) and solve for \( t \)
\[
\frac{1}{2} Q_0 = Q_0 e^{-\frac{t}{RC}} \quad \Rightarrow \quad 0.5
\]
\[
\ln \frac{1}{2} = -\frac{t}{RC} \quad \Rightarrow \quad t = -RC \ln \frac{1}{2} = RC \ln 2 = 0.69 \, RC
\]
CHARGING: THROUGH A RESISTOR

Kirchhoff's Voltage Rule

\[ E - IR - \frac{Q}{C} = 0 \]

- Initial condition: \( Q = 0 \Rightarrow I = \frac{E}{R} \)
- Charge builds up on capacitor
- Eventually it's fully charged: \( I = 0 \quad Q_f = E \cdot C \)

Square wave generator

\[ Q(t) = Q_f \left(1 - e^{-t/RC}\right) \]

MAGNETISM: We will now start to learn some things about magnetism and magnetic forces. Magnetism has been known for a very long time (thousands of years) because there are materials in nature that are naturally magnetic.

MAGNETIC FORCES: It's easy to demonstrate that there are magnetic forces by playing with any kind of magnet. For example, we can deflect compass needles, pick up nails etc.

The actual force laws in these situations are pretty complicated. If we play with bar magnets things are somewhat simpler.

The first thing we might notice is that the two ends are different. One end attracts the "N" pole of our compass needle and the other attracts the "S" pole.

Procedure: take several bar magnets & paint the ends
red and blue to remember which end is which. Then we can study the force laws. Here are examples:

1. repel
2. attract
3. attract

After doing a variety of experiments one learns that the force law can be summarized by saying that like poles repel and unlike poles attract. At some point you might also realize that the compass needles are really just smaller bar magnets.

So we could say that bar magnets behave much like what we refer to as "electric dipoles":

\[ + - \]

\[ - + \]

etc.

There are, however, important differences. Suppose we take a bar magnet and cut it in half to get an isolated N-pole. The result is two shorter bar magnets - except for being short, just like the original one. There is no way to make an isolated N (or S) pole.

**Magnetic Fields:** If we carry a compass needle around a bar magnet we get the idea that there is some kind of force field - analogous in some ways to the electric field \( \mathbf{E} \). Let's start by defining a procedure to "measure" the magnetic field, \( \mathbf{B} \).

To find \( \mathbf{B} \) at any point in space, place a
small compass needle there. Then, by definition, the magnetic field points along the compass needle with the N pole of the needle giving the direction of $B$.

Notice that if we could make an electric dipole (and keep the charge in place) we could find electric fields the same way.

FIELD OF A BAR MAGNET:

![Diagram of a bar magnet showing magnetic field lines]

We can also "picture" the magnetic field with iron filings. FINE.

MAGNETIC FIELD AROUND A WIRE: In 1820 Hans Oersted made a chance discovery that eventually led to a much more complete understanding of magnetism. What Oersted found is that he could produce a magnetic field (i.e. deflect a compass needle) by passing current through a wire.

Electric current (moving charges) make a mag. field => we see a connection for the 1st time between electricity and magnetism.
What does the magnetic field look like?

Map out the field => we find that the magnetic field lines are circles. This is now totally different from anything we saw with electric fields. With \( \vec{E} \), the lines start one place (on + charges) and end somewhere else (-charges or maybe infinity). Here the lines loop around and end on themselves.

**DIRECTION OF \( \vec{B} \):** Remembering the direction of \( \vec{B} \) is somewhat tricky since it’s \( \perp \) to the direction of current flow.

**RIGHT HAND RULE** (#2 in text)
- thumb in direction of I
- curl fingers
- fingers point the direction of field circles.

**SUMMARY:** We now see that there are two ways to produce a magnetic field - permanent magnets and wires carrying current. Of these, the second is much easier to understand and that’s what we will focus on.