23.22 For a plane refracting surface \((R \to \infty)\)

\[
\frac{n_2}{p} + \frac{n_2 - n_1}{q} = \frac{n_2 - n_1}{R} \quad \text{becomes} \quad q = -\frac{n_2}{n_1} p
\]

(a) When the pool is full, \(p = 2.00 \text{ m}\) and

\[
q = -\left(\frac{1.00}{1.333}\right)(2.00 \text{ m}) = -1.50 \text{ m}
\]

or the pool appears to be \([1.50 \text{ m}]\) deep.

(b) If the pool is half filled, then \(p = 1.00 \text{ m}\) and \(q = -0.750 \text{ m}\). Thus, the bottom of the pool appears to be 0.75 m below the water surface or \([1.75 \text{ m}]\) below ground level.

23.23 Since the center of curvature of the surface is on the side the light comes from, \(R < 0\)
giving \(R = -4.0 \text{ cm}\). Then, \(\frac{n_1}{p} + \frac{n_2}{q} = -\frac{n_2 - n_1}{R}\) becomes

\[
\frac{1.00}{q} = \frac{1.00 - 1.50}{-4.0 \text{ cm}} = \frac{150}{4.0 \text{ cm}}, \quad \text{or} \quad q = -4.0 \text{ cm}
\]

Thus, the magnification \(M = \frac{h'}{h} = -\left(\frac{n_1}{n_2}\right) \frac{q}{p}\), gives

\[
h' = -\left(\frac{n_2 q}{n_2 p}\right) h = -\frac{150(-4.0 \text{ cm})}{1.00(4.0 \text{ cm})}(2.5 \text{ mm}) = [3.8 \text{ mm}]\]
23.29 From the thin lens equation, \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \), the image distance is found to be

\[
q = \frac{fp}{p-f} = \frac{(20.0 \text{ cm})p}{p-20.0 \text{ cm}}
\]

(a) If \( p = 40.0 \text{ cm} \), then \( q = 40.0 \text{ cm} \) and \( M = \frac{q}{p} = \frac{40.0 \text{ cm}}{40.0 \text{ cm}} = 1.00 \)

The image is real, inverted, and 40.0 cm beyond the lens.

(b) If \( p = 20.0 \text{ cm} \), \( q \to \infty \) No image formed. Parallel rays leave the lens.

(c) When \( p = 10.0 \text{ cm} \), \( q = -20.0 \text{ cm} \) and

\[
M = \frac{q}{p} = \frac{(-20.0 \text{ cm})}{10.0 \text{ cm}} = -2.00
\]

The image is virtual, upright, and 20.0 cm in front of the lens.

23.31 From the thin lens equation, \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \), the image distance is found to be

\[
q = \frac{fp}{p-f} = \frac{(-20.0 \text{ cm})p}{p-(-20.0 \text{ cm})} = \frac{(20.0 \text{ cm})p}{p+20.0 \text{ cm}}
\]

(a) If \( p = 40.0 \text{ cm} \), then \( q = -13.3 \text{ cm} \) and \( M = \frac{q}{p} = \frac{-13.3 \text{ cm}}{40.0 \text{ cm}} = -1/3 \)

The image is virtual, upright, and 13.3 cm in front of the lens.

(b) If \( p = 20.0 \text{ cm} \), then \( q = -10.0 \text{ cm} \) and

\[
M = \frac{q}{p} = \frac{-10.0 \text{ cm}}{20.0 \text{ cm}} = -1/2
\]

The image is virtual, upright, and 10.0 cm in front of the lens.

(c) When \( p = 10.0 \text{ cm} \), \( q = -6.67 \text{ cm} \) and \( M = \frac{q}{p} = \frac{(-6.67 \text{ cm})}{10.0 \text{ cm}} = -2/3 \)

The image is virtual, upright, and 6.67 cm in front of the lens.
23.34 We must first realize that we are looking at an upright, magnified, virtual image. Thus, we have a real object located between a converging lens and its front-side focal point, so \( q < 0, \, p > 0, \) and \( f > 0 \).

The magnification is \( M = -\frac{q}{p} = +2 \), giving \( q = -2p \). Then, from the thin lens equation,

\[
\frac{1}{p} - \frac{1}{2p} = +\frac{1}{2p} = \frac{1}{f} \quad \text{or} \quad f = 2p = 2(2.84 \text{ cm}) = 5.68 \text{ cm}
\]

23.40 With \( p_1 = 20.0 \text{ cm} \) and \( f_1 = 25.0 \text{ cm} \), the thin lens equation gives the position of the image formed by the first lens as

\[
q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(20.0 \text{ cm})(25.0 \text{ cm})}{20.0 \text{ cm} - 25.0 \text{ cm}} = -100 \text{ cm}
\]

and the magnification by this lens is \( M_1 = -\frac{q_1}{p_1} = -\frac{(-100 \text{ cm})}{20.0 \text{ cm}} = +5.00 \).

This virtual image serves as the object for the second lens, so the object distance is \( p_2 = 25.0 \text{ cm} + |q_1| = 125 \text{ cm} \). Then, the thin lens equation gives the final image position as

\[
q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(125 \text{ cm})(-10.0 \text{ cm})}{125 \text{ cm} - (-10.0 \text{ cm})} = -9.26 \text{ cm}
\]

with a magnification by the second lens of

\[
M_2 = -\frac{q_2}{p_2} = -\frac{(-9.26 \text{ cm})}{125 \text{ cm}} = +0.0741
\]

Thus, the final image is located \( 9.26 \text{ cm} \) in front of the second lens and the overall magnification is \( M = M_1 M_2 = (+5.00)(+0.0741) = +0.370 \).
23.50 Since the object is midway between the lens and mirror, the object distance for the mirror is \( p_1 = +12.5 \text{ cm} \). The mirror equation gives the image position as

\[
\frac{1}{q_1} = \frac{2}{R} \frac{1}{p_1} = \frac{2}{20.0 \text{ cm}} \frac{1}{12.5 \text{ cm}} = \frac{5-4}{50.0 \text{ cm}} = \frac{1}{50.0 \text{ cm}}, \text{ or } q_1 = +50.0 \text{ cm}
\]

This image serves as the object for the lens, so \( p_2 = 25.0 \text{ cm} - q_1 = -25.0 \text{ cm} \). Note that since \( p_2 < 0 \), this is a virtual object. The thin lens equation gives the image position for the lens as

\[
q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-25.0 \text{ cm})(-16.7 \text{ cm})}{-25.0 \text{ cm} - (-16.7 \text{ cm})} = -50.3 \text{ cm}
\]

Since \( q_2 < 0 \), this is a virtual image that is located 50.3 cm in front of the lens or 25.3 cm behind the mirror. The overall magnification is

\[
M = M_1 M_2 = \left( \frac{q_1}{p_1} \right) \left( \frac{-q_2}{p_2} \right) = \left( \frac{50.0 \text{ cm}}{12.5 \text{ cm}} \right) \left[ \frac{(-50.3 \text{ cm})}{(-25.0 \text{ cm})} \right] = +8.05
\]

Since \( M > 0 \), the final image is upright.

24.3 (a) The distance between the central maximum and the first order bright fringe is

\[
\Delta y = y_{\text{bright}} \bigg|_{m=1} - y_{\text{bright}} \bigg|_{m=0} = \frac{\lambda L}{d}, \text{ or}
\]

\[
\Delta y = \frac{\lambda L}{d} = \frac{(546.1 \times 10^{-9} \text{ m})(1.20 \text{ m})}{0.250 \times 10^{-3} \text{ m}} = 2.62 \times 10^{-3} \text{ m} = 2.62 \text{ mm}
\]

(b) The distance between the first and second dark bands is

\[
\Delta y = y_{\text{dark}} \bigg|_{m=1} - y_{\text{dark}} \bigg|_{m=0} = \frac{\lambda L}{d} = 2.62 \text{ mm} \text{ as in (a) above.}
\]
24.8 In a double-slit interference pattern the distance from the central maximum to the position of the $m^{th}$ order bright fringe is given by

$$y_m = m \left( \frac{\lambda L}{d} \right)$$

where $d$ is the distance between the splits and $L$ is the distance to the screen. Thus, the spacing between the first- and second-order bright fringes is

$$\Delta y = y_2 - y_1 = (2-1) \left( \frac{\lambda L}{d} \right) = 1 \left[ \left( \frac{600 \times 10^{-6} \text{ m}}{0.050 \times 10^{-3} \text{ m}} \right) \left( \frac{2.50 \text{ m}}{600 \times 10^{-6} \text{ m}} \right) \right] = 0.030 \text{ m} = 3.0 \text{ cm}$$

24.10 The angular deviation from the line of the central maximum is given by

$$\theta = \tan^{-1} \left( \frac{y}{L} \right) = \tan^{-1} \left( \frac{1.80 \text{ cm}}{140 \text{ cm}} \right) = 0.737^\circ$$

(a) The path difference is then

$$\delta = d \sin \theta = (0.150 \text{ mm}) \sin (0.737^\circ) = 1.93 \times 10^{-3} \text{ mm} = 1.93 \mu\text{m}$$

(b) $$\delta = (1.93 \times 10^{-6} \text{ m}) \left( \frac{\lambda}{643 \times 10^{-9} \text{ m}} \right) = 3.0 \lambda$$

(c) Since the path difference for this position is a whole number of wavelengths, the waves interfere constructively and produce a maximum at this spot.

24.12 The path difference in the two waves received at the home is $\delta = 2d - 30.0 \text{ km}$ where $d$ is defined in the figure at the right. For minimum cloud height and (hence minimum path difference) to yield destructive interference, $\delta = \lambda/2$ giving

$$d_{\text{min}} = \frac{1}{2} \left( 30.0 \text{ km} + \frac{\lambda}{2} \right) = 15.1 \text{ km}, \text{ and}$$

$$h_{\text{min}} = \sqrt{d_{\text{min}}^2 - (15.0 \text{ km})^2} = \sqrt{(15.1 \text{ km})^2 - (15.0 \text{ km})^2} = 1.73 \text{ km}$$
24.15 Light reflecting from the upper surface undergoes phase reversal while that reflecting from the lower surface does not. The condition for constructive interference in the reflected light is then

\[ 2t = \frac{\lambda_n}{2} = m\lambda_n, \text{ or } t = \left(m + \frac{1}{2}\right)\frac{\lambda_n}{2} = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_{\text{film}}}, \text{ } m = 0, 1, 2, \ldots \]

For minimum thickness, \( m = 0 \) giving

\[ t = \frac{\lambda}{4n_{\text{film}}} = \frac{500 \text{ nm}}{4(1.36)} = 91.9 \text{ nm} \]

24.16 With \( n_{\text{glass}} > n_{\text{air}} \) and \( n_{\text{liquid}} < n_{\text{glass}} \), light reflecting from the air-glass boundary experiences a 180° phase shift, but light reflecting from the glass-liquid boundary experiences no shift. Thus, the condition for destructive interference in the two reflected waves is

\[ 2n_{\text{glass}}t = m\lambda \quad \text{where} \quad m = 0, 1, 2, \ldots \]

For minimum (non-zero) thickness, \( m = 1 \) giving

\[ t = \frac{\lambda}{2n_{\text{glass}}} = \frac{580 \text{ nm}}{2(1.50)} = 193 \text{ nm} \]

24.20 The transmitted light is brightest when the reflected light is a minimum (that is, the same conditions that produce destructive interference in the reflected light will produce constructive interference in the transmitted light). As light enters the air layer from glass, any light reflected at this surface has zero phase change. Light reflected from the other surface of the air layer (where light is going from air into glass) does have a phase reversal. Thus, the condition for destructive interference in the light reflected from the air film is \( 2t = m\lambda_n, m = 0, 1, 2, \ldots \)

Since \( \lambda_n = \frac{\lambda}{n_{\text{film}}} = \frac{\lambda}{1.00} = \lambda \), the minimum non-zero plate separation satisfying this condition is

\[ d = t = (1)\frac{\lambda}{2} = \frac{580 \text{ nm}}{2} = 290 \text{ nm} \]
24.22  Light reflecting from the lower surface of the air layer experiences phase reversal, but light reflecting from the upper surface of the layer does not. The requirement for a dark fringe (destructive interference) is then

\[ 2t = m\lambda_n = m\left(\frac{\lambda}{n_{air}}\right) = m\lambda, \text{ where } m = 0, 1, 2, \ldots \]

At the thickest part of the film \((t = 2.00 \mu m)\), the order number is

\[ m = \frac{2t}{\lambda} = \frac{2(2.00 \times 10^{-6} \text{ m})}{546.1 \times 10^{-9} \text{ m}} = 7.32 \]

Since \(m\) must be an integer, \(m = 7\) is the order of the last dark fringe seen. Counting the \(m = 0\) order along the edge of contact, a total of 8 dark fringes will be seen.

24.29  The distance on the screen from the center to either edge of the central maximum is

\[ y = L \tan \theta \approx L \sin \theta = L\left(\frac{\lambda}{a}\right) \]

\[ = (1.00 \text{ m})\left(\frac{632.8 \times 10^{-9} \text{ m}}{0.300 \times 10^{-3} \text{ m}}\right) = 2.11 \times 10^{-3} \text{ m} = 2.11 \text{ mm} \]

The full width of the central maximum on the screen is then

\[ 2y = 4.22 \text{ mm} \]

24.30  \(a\)  Dark bands occur where \(\sin \theta = m(\lambda/a)\). At the first dark band, \(m = 1\), and the distance from the center of the central maximum is

\[ y_1 = L \tan \theta \approx L \sin \theta = L\left(\frac{\lambda}{a}\right) \]

\[ = (1.5 \text{ m})\left(\frac{600 \times 10^{-9} \text{ m}}{0.40 \times 10^{-3} \text{ m}}\right) = 2.25 \times 10^{-3} \text{ m} = 2.3 \text{ mm} \]

\(b\)  The width of the central maximum is \(2y_1 = 2(2.25 \text{ mm}) = 4.5 \text{ mm} \)