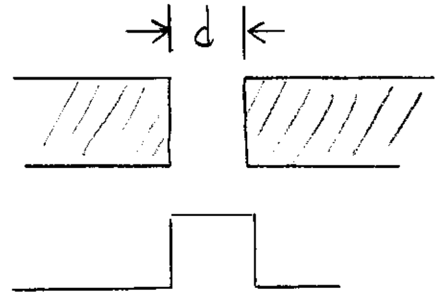
1) Find the deBroglie wavelength of an electron with kinetic energy of 15 eV.

$$E = \frac{p^2}{2m} \qquad p = \sqrt{2mE}$$

$$\lambda = \frac{h}{p} = \frac{hc}{[2Emc^2]^{\frac{1}{2}}} = \frac{1240\,eV\cdot nm}{[2(5.11\times10^5 eV)(15 eV)]^{\frac{1}{2}}} = 0.317\,nm$$

$$\boxed{0.317\ nm.}$$

2) The drawing below shows two copper electrodes separated by a small distance, $d = 0.5$ nm. Make a rough estimate of the probability that an electron incident from the left could tunnel across the gap between the electrodes. The work function of copper (*i.e.* the minimum energy required to remove an electron from the surface) is 4.7 eV.



$T \sim e^{-2\alpha L}$ where

$$\alpha = \left[\frac{2m}{\hbar^2}(V_0 - E)\right]^{\frac{1}{2}}$$

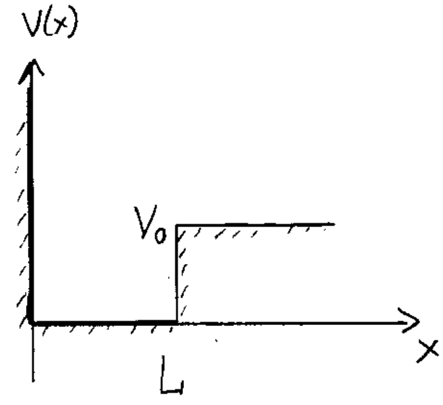
The electrons require 4.7 eV to be free, so $V_0 - E = 4.7\,eV$

$$2\alpha L = 2\,\frac{[2(5.11\times10^5 eV)(4.7 eV)]^{\frac{1}{2}}}{(1240\,eV\cdot nm/2\pi)} \cdot 0.5\,nm = 11.1$$

$$T \sim e^{-11.1} = 1.50\times10^{-5}$$

$$\boxed{1.50\times10^{-5}}$$

3) When a particle of mass m is confined in the semi-infinite square well shown at the right, it's wave function will be of the form



$$\psi(x) = \begin{cases} A\sin kx & \text{for } 0 < x < L \\ Ce^{-\alpha x} & \text{for } x > L \end{cases}$$

Suppose that for $L = 5\,\text{nm}$, one of the bound states has $k = 1.7/\text{nm}$.

(a) Find the energy of the state.

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V\psi = E\psi \qquad \text{For } 0<x<L \quad V=0 \text{ so}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}A\sin kx = EA\sin kx$$

$$+\frac{\hbar^2k^2}{2m}A\sin kx = EA\sin kx$$

$$E = \frac{\hbar^2k^2}{2m}$$

$$\boxed{\hbar^2k^2/2m}$$

(b) Write down the matching conditions that the wave function must satisfy at $x = L$.

ψ: $\quad A\sin kL = Ce^{-\alpha L}$

ψ': $\quad kA\cos kL = -\alpha Ce^{-\alpha L}$

(c) Use the matching conditions to find α. Use $Ce^{-\alpha L} = A\sin kL$ + sub. into the second equation

$$k\cancel{A}\cos kL = -\alpha \cancel{A}\sin kL$$

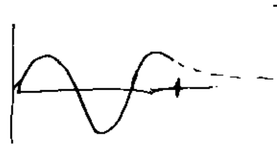
$$\alpha = -k\frac{\cos kL}{\sin kL} = -1.7\cot(5\times 1.7)$$

$$= 1.282/\text{nm}$$

$$\boxed{1.282/\text{nm}}$$

(d) Determine which state this is; *i.e.*, find the value of n where $n = 1$ for the ground state, $n = 2$ for the first excited state, *etc.* You need to explain or show some work to get credit.

Plotting $A\sin kx$ from 0 to L $\Rightarrow$ $\sin y$ from 0 to 8.5 radians
0 to 2.7π



Two nodes $\Rightarrow$
$n=3$

$$\boxed{n=3}$$

4) An electron in the hydrogen atom is in the 3d state.

(a) Find the orbital angular momentum of the electron (in units of $\hbar$).

$\ell = 2$

$$L = \sqrt{\ell(\ell+1)}\ \hbar$$

$\boxed{\sqrt{6}\ \hbar}$

(b) Find the energy of the electron (in eV).

$$E = -\frac{13.6\,\text{eV}}{n^2} = -\frac{13.6\,\text{eV}}{9}$$

$$= -1.51\,\text{eV}$$

$\boxed{-1.51\,\text{eV}}$

(c) Ignoring electron spin, find the total number of quantum states that have this same energy.

3s, 3p, 3d are all degenerate

(1) + (3) + (5)

$\boxed{9}$

(d) Make a table listing the quantum numbers n, ℓ and m_ℓ of each of the degenerate states. Indicate the spectroscopic label (*e.g.* 1s, 2s etc) that would apply to each state.

n	ℓ	m_ℓ	Label
3	0	0	3s
3	1	1	3p
3	1	0	3p
3	1	-1	3p
3	2	2	3d
3	2	1	3d
3	2	0	3d
3	2	-1	3d
3	2	-2	3d

5) The wave function for a hydrogen atom in the ground state can be written in the form

$$\psi(r) = R(r)\, Y_0^0(\theta,\phi) \qquad \text{where} \qquad R(r) = Ce^{-r/a_0} \qquad \text{and} \qquad Y_0^0 = \frac{1}{\sqrt{4\pi}}.$$

Working from this information, calculate the expectation value of the potential energy V. The desired result is a formula that involves only known constants. Remember that $V(r) = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r}$. The following integral will be useful: $\int_0^\infty x^n e^{-x}\,dx = n!$

$$P(r) = r^2|R(r)|^2 = C^2 r^2 e^{-2r/a_0}$$

Normalize to find C^2

$$\int_0^\infty P(r)\,dr = 1 = C^2\int_0^\infty r^2 e^{-2r/a_0}\,dr \qquad x = 2r/a_0$$

$$r = \frac{a_0}{2}x$$

$$1 = C^2\left(\frac{a_0}{2}\right)^3\int_0^\infty x^2 e^{-x}\,dx \qquad dr = \left(\frac{a_0}{2}\right)dx$$

$$= C^2\left(\frac{a_0}{2}\right)^3(2) \qquad C^2 = \frac{4}{a_0^3}$$

$$\langle V\rangle = -\frac{e^2}{4\pi\epsilon_0}\int_0^\infty \frac{1}{r}P(r)\,dr = -\frac{e^2}{4\pi\epsilon_0}\int C^2 r e^{-2r/a_0}\,dr$$

$$= -\frac{e^2}{4\pi\epsilon_0}\left(\frac{4}{a_0^3}\right)\left(\frac{a_0}{2}\right)^2\int_0^\infty x e^{-x}\,dx$$

$$= -\frac{e^2}{4\pi\epsilon_0}\cdot\frac{1}{a_0}(1)$$

$$\boxed{-\frac{e^2}{4\pi\epsilon_0}\cdot\frac{1}{a_0}}$$