Homework I Solutions

1-4 (a) In all cases one wants the speed of the plane relative to the ground. For the upwind and downwind legs, where v' in the figure is given by $\left(c^2 - v^2\right)^{1/2}$

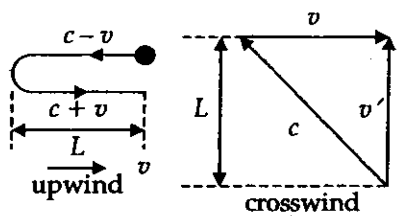
$$t_{u+d} = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2L}{c}\left(\frac{1}{1 - v^2/c^2}\right).$$

For the crosswind case, the plane's speed along L is $v' = \left(c^2 - v^2\right)^{1/2}$

$$t_c = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c}\frac{1}{\sqrt{1 - (v/c)^2}}$$

$$t_{u+d} = \frac{2(100\ \text{mi})}{500\ \text{mi/h}}\left(\frac{1}{1 - (100)^2/(500)^2}\right) = 0.416\,7\ \text{h}$$

$$t_c = \frac{2(100\ \text{mi})}{500\ \text{mi/h}}\left(\frac{1}{\sqrt{0.96}}\right) = 0.408\,2\ \text{h}$$



(b) $\Delta t = t_{u+d} - t_c = 0.008\,5\ \text{h} \cong 0.009\ \text{h}$ or $0.510\ \text{min} \cong 0.5\ \text{min}$

1-5 This is a case of dilation. $T = \gamma T'$ in this problem with the proper time $T' = T_0$

$$T = \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} T_0 \Rightarrow \frac{v}{c} = \left[1 - \left(\frac{T_0}{T}\right)^2\right]^{1/2};$$

in this case $T = 2T_0$, $v = \left\{1 - \left[\frac{L_0/2}{L_0}\right]^2\right\}^{1/2} = \left[1 - \left(\frac{1}{4}\right)\right]^{1/2}$ therefore $v = 0.866c$.

1-7 The problem is solved by using time dilation. This is also a case of $v \ll c$ so the binomial expansion is used $\Delta t = \gamma \Delta t' \cong \left[1 + \frac{v^2}{2c^2}\right]\Delta t'$, $\Delta t - \Delta t' = \frac{v^2 \Delta t'}{2c^2}$; $v = \left[\frac{2c^2(\Delta t - \Delta t')}{\Delta t'}\right]^{1/2}$;
$\Delta t = (24\ \text{h/day})(3\,600\ \text{s/h}) = 86\,400\ \text{s}$; $\Delta t = \Delta t' - 1 = 86\,399\ \text{s}$;

$$v = \left[\frac{2(86\,400\ \text{s} - 86\,399\ \text{s})}{86\,399\ \text{s}}\right]^{1/2} = 0.004\,8c = 1.44 \times 10^6\ \text{m/s}.$$

1-10 (a) $\tau = \gamma \tau'$ where $\beta = \frac{v}{c}$ and

$$\gamma =' \left(1 - \beta^2\right)^{-1/2} = \tau'\left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \left(2.6 \times 10^{-8}\ \text{s}\right)\left[1 - (0.95)^2\right]^{-1/2} = 8.33 \times 10^{-8}\ \text{s}$$

(b) $d = v\tau = (0.95)(3 \times 10^8)(8.33 \times 10^8\ \text{s}) = 24\ \text{m}$

1-12 (a) 70 beats/min or $\Delta t' = \frac{1}{70}$ min

(b) $\Delta t = \gamma \Delta t' = \left[1-(0.9)^2\right]^{-1/2}\left(\frac{1}{70}\right)$ min $= 0.032\,8$ min/beat or the number of beats per minute $\approx 30.5 \approx 31$.

1-16 For an observer approaching a light source, $\lambda_{ob} = \left[\frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}}\right]\lambda_{source}$. Setting $\beta = \frac{v}{c}$ and after some algebra we find,

$$\beta = \frac{\lambda_{source}^2 - \lambda_{obs}^2}{\lambda_{source}^2 + \lambda_{obs}^2} = \frac{(650\text{ nm})^2 - (550\text{ nm})^2}{(650\text{ nm})^2 + (550\text{ nm})^2} = 0.166$$

$$v = 0.166c = \left(4.98\times10^7\text{ m/s}\right)\left(2.237\text{ mi/h}\right)\left(\text{m/s}\right)^{-1} = 1.11\times10^8\text{ mi/h}.$$

1-32 The spacecraft's speed in the Earth's reference frame is: $v = \frac{20\text{ light-hours}}{25\text{ hours}} = 0.8c$,

$$\gamma = \left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2} = \frac{1}{\left[1-(0.8)^2\right]^{1/2}} = 1.67.$$

The spacecraft's clocks tick through $\Delta t' = \frac{\Delta t}{\gamma} = \frac{25\text{ h}}{1.67} = 15.0\text{ h}$, ten hours less than in the Earth's frame.