Homework 10 Solutions

9-1 $\Delta E = 2\mu_B B = hf$

$2(9.27\times10^{-24}\ \text{J/T})(0.35\ \text{T}) = (6.63\times10^{-34}\ \text{Js})f$ so $f = 9.79\times10^{9}$ Hz

9-4 (a) $3d$ subshell $\Rightarrow l=2 \Rightarrow m_l = -2, -1, 0, 1, 2$ and $m_s = \pm\frac{1}{2}$ for each m_l

n	l	m_l	m_s
3	2	−2	−1/2
3	2	−2	+1/2
3	2	−1	−1/2
3	2	−1	+1/2
3	2	0	−1/2
3	2	0	+1/2
3	2	1	−1/2
3	2	1	+1/2
3	2	2	−1/2
3	2	2	+1/2

(b) $3p$ subshell: for a p state, $l=1$. Thus m_l can take on values $-l$ to l, or −1, 0, 1. For each m_l, m_s can be $\pm\frac{1}{2}$.

n	l	m_l	m_s
3	1	−1	−1/2
3	1	−1	+1/2
3	1	0	−1/2
3	1	0	+1/2
3	1	1	−1/2
3	1	1	+1/2

9-9 With $s=\frac{3}{2}$, the spin magnitude is $|S| = [s(s+1)]^{1/2}\hbar = \left(\frac{[15]^{1/2}}{2}\right)\hbar$. The z-component of spin is $S_z = m_s\hbar$ where m_s ranges from $-s$ to s in integer steps or, in this case, $m_s = -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}$. The spin vector S is inclined to the z-axis by an angle θ such that

$$\cos(\theta) = \frac{S_z}{|S|} = \frac{m_s\hbar}{([15]^{1/2}/2)\hbar} = \frac{m_s}{[15]^{1/2}/2} = -\frac{3}{(15)^{1/2}}, -\frac{1}{(15)^{1/2}}, +\frac{1}{(15)^{1/2}}, +\frac{3}{(15)^{1/2}}$$

or $\theta = 140.8°, 105.0°, 75.0°, 39.2°$. The Ω^- *does* obey the Pauli Exclusion Principle, since the spin s of this particle is half-integral, as it is for all fermions.

9-11 For a d electron, $l=2$; $s=\frac{1}{2}$; $j = 2+\frac{1}{2}, 2-\frac{1}{2}$

For $j=\frac{5}{2}$; $m_j = -\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$

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9-13 (a) $4F_{5/2} \rightarrow n=4,\ l=3,\ j=\frac{5}{2}$

(b) $|J|=[j(j+1)]^{1/2}\hbar=\left[\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)\right]^{1/2}\hbar=\left[\frac{35}{4}\right]^{1/2}\hbar=\left[\frac{(35)^{1/2}}{2}\right]\hbar$

(c) $J_z=m_j\hbar$ where m_j can be $-j,\ -j+1,\ \ldots,\ j-1,\ j$ so here m_j can be $-\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. J_z can be $-\frac{5}{2}\hbar, -\frac{3}{2}\hbar, -\frac{1}{2}\hbar, \frac{1}{2}\hbar, \frac{3}{2}\hbar$, or $\frac{5}{2}\hbar$.

9-21 (a) $1s^2 2s^2 2p^4$

(b) For the two 1s electrons, $n=1,\ \ l=0,\ m_l=0,\ m_s=\pm\frac{1}{2}$.

For the two 2s electrons, $n=2,\ l=0,\ m_l=0,\ m_s=\pm\frac{1}{2}$.

For the four 2p electrons, $n=2,\ l=1,\ m_l=1,\ 0,\ -1,\ m_s=\pm\frac{1}{2}$.

11-5 (a) The separation between two adjacent rotationally levels is given by $\Delta E=\left(\frac{\hbar^2}{I}\right)l$, where l is the quantum number of the higher level. Therefore

$$\Delta E_{10}=\frac{\Delta E_{65}}{6}$$

$$\lambda_{10}=6\lambda_{65}=6(1.35\text{ cm})=8.10\text{ cm}$$

$$f_{10}=\frac{c}{\lambda_{10}}=\frac{3.00\times10^{10}\text{ cm/s}}{8.10\text{ cm}}=3.70\text{ GHz}$$

(b) $\Delta E_{10}=hf_{10}=\frac{\hbar^2}{I}$;

$$I=\frac{\hbar}{2\pi f_{10}}=\frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{(2\pi)(3.70\times10^{9}\text{ Hz})}$$

$$I=4.53\times10^{-45}\text{ kg}\cdot\text{m}^2$$

W20) (a) The excitation energy is $E = \ell(\ell+1)\frac{\hbar^2}{2I}$ where $I=\mu R_0^2$

For CO the reduced mass is

$$\mu=\frac{m_1 m_2}{m_1+m_2}=\frac{(12u)\cdot(16u)}{12u+16u}=6.857\,u$$

Here

$$1u = 931.5\text{ MeV}/c^2 = 9.315\times10^8\text{ eV}/c^2$$

So $\quad 1u\cdot c^2 = 9.315\times10^8\text{ eV}$

$$\frac{\hbar^2}{2I}=\frac{(\hbar c)^2}{2\mu c^2\cdot R_0^2}=\left(\frac{1}{2\pi}\right)^2\frac{(hc)^2}{2\mu c^2 R_0^2}=\left(\frac{1}{2\pi}\right)^2\frac{(1240\text{ eV}\cdot\text{nm})^2}{2(6.857)(9.315\times10^8\text{ eV})(.113\text{ nm})^2}$$

$= 2.388 \times 10^{-4} eV$

Thus for $\ell = 1$

$$E_1 = (1)(2)\frac{\hbar^2}{2I} \Rightarrow \boxed{E_1 = 4.775 \times 10^{-4} eV}$$

$$E_2 = (2)(3)\frac{\hbar^2}{2I} \Rightarrow \boxed{E_2 = 1.433 \times 10^{-3} eV}$$

(b) We can find N_ℓ / N_0 using the Boltzmann formula. ~~the ~~ ~~distribution~~ Here

$$kT = (8.617 \times 10^{-5} eV/K)(290 K) = 0.02499 \, eV$$

Thus

$$\frac{N_1}{N_0} = \frac{g_1}{g_0} e^{-E_1/kT} = (3) e^{-4.775 \times 10^{-4} eV / 2.499 \times 10^{-2} eV} \Rightarrow \boxed{\frac{N_1}{N_0} = 2.94}$$

$$\frac{N_2}{N_1} = \frac{g_2}{g_0} e^{-E_2/kT} = (5) e^{-1.433 \times 10^{-3} eV / 2.499 \times 10^{-2} eV} \Rightarrow \boxed{\frac{N_2}{N_0} = 4.72}$$

W21). The absorbed photons have $\lambda = 3.69$ mm and therefore the energy is

$$E_\gamma = hf = hc/\lambda = (1240 \, eV \cdot nm)/(3.69 \times 10^6 nm) = 3.36 \times 10^{-4} eV.$$

Since

$$E_{rot} = \frac{\ell(\ell+1)\hbar^2}{2I}$$

the energy for the $\ell = 0$ to $\ell = 1$ transition is

$$E = E_1 - E_0 = \frac{(1)(2)\hbar^2}{2I} - 0 = \frac{\hbar^2}{I}.$$

Thus

$$E_\gamma = 3.36 \times 10^{-4} eV = \frac{\hbar^2}{I} \Rightarrow I = \hbar^2 / E_\gamma$$

but

$$I = \mu R_0^2 \quad \text{so} \quad R_0^2 = \hbar^2/\mu E_\gamma = (\hbar c)^2 / \mu c^2 E_\gamma .$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(7)(19)}{7+19} u = 5.12 \, u \Rightarrow \mu c^2 = (5.12) \cdot (931.5 \, MeV) = 4.76 \times 10^9 e$$

Thus

$$R_0 = (\hbar c) / [\mu c^2 E_\gamma]^{1/2} = \left(\frac{1}{2\pi}\right)(1240 \, eV \cdot nm) / [(4.76 \times 10^9 eV)(3.36 \times 10^{-4} eV)]$$

$$\Rightarrow \boxed{R_0 = 0.156 \, nm}$$

W-22) The vibrational energy is $E_{vib} = (\nu + \frac{1}{2})\hbar\omega_o$ so the energy for $\nu=0$ to $\nu=1$ is

$$E_\gamma = \tfrac{3}{2}\hbar\omega_o - \tfrac{1}{2}\hbar\omega_o = \hbar\omega_o$$

but $E_\gamma = hf$ so

$$f = \hbar\omega_o / h = \omega_o / 2\pi \Rightarrow \omega_o = 2\pi f$$

$\Rightarrow$

$$\omega_o = (2\pi)(5.63\times10^{13}/s) \Rightarrow \omega_o = 3.54\times10^{14}/s$$

(a) To find k we use $\omega_o = \sqrt{\frac{k}{\mu}} \Rightarrow k = \mu\omega_o^2$

$$\mu = \frac{m_1 m_2}{m_1+m_2} = \frac{(14u)(16u)}{14u+16u} = 7.47\,u = (7.47)\cdot(1.66\times10^{-27}\,kg) = 1.24\times10^{-26}\,kg$$

so

$$k = \mu\omega_o^2 = (1.24\times10^{-26}\,kg)(3.54\times10^{14}/s)^2 \Rightarrow \boxed{k = 1551\ N/m}$$

(b) In the ground state the energy is $E_o = \frac{1}{2}\hbar\omega_o$. To find the amplitude of the vibrational motion, A, we need to recall that $E = K+V = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$. When x is at it's maximum value the kinetic energy is zero (the maximum x is the turning point so v must be zero) $\Rightarrow$

$$\tfrac{1}{2}kA^2 = E = \tfrac{1}{2}\hbar\omega_o$$

Thus

$$A = [\hbar\omega_o/k]^{1/2}$$

$$A = [(1.055\times10^{-34}\,J\cdot s)(3.54\times10^{14}/s)\,/\,(1551\,N/m)]^{1/2} = 4.91\times10^{-12}\,m$$

$$\Rightarrow \boxed{A = 4.91\times10^{-3}\ nm}$$