Homework 11 Solutions

10-14 (a) $E_F = 7.05\text{ eV}$ at 0 K for copper.

$$E_{av} = \frac{3}{5}E_F = \frac{3}{5}(7.05\text{ eV}) = 4.23\text{ eV}$$

(b) $E_{av}(\text{per molecule}) = \frac{3}{2}k_B T = 4.23\text{ eV}$

$$T = \frac{(4.23\text{ eV})(1.6\times10^{-19}\text{ J/eV})}{(1.5)(1.38\times10^{-23}\text{ J/K})} = 3.27\times10^{4}\text{ K} = 32\,700\text{ K}$$

10-21 $\rho = 0.971\text{ g/cm}^3$, $M = 23.0\text{ g/mole}$ (sodium)

(a) $n = \dfrac{N_A \rho}{M}$

$$n = (6.02\times10^{23}\text{ electrons/mole})(0.971\text{ g/cm}^3)(23.0\text{ g/mole})$$

$$n = 2.54\times10^{22}\text{ electrons/cm}^3 = 2.54\times10^{28}\text{ electrons/m}^3$$

(b) $E_F = \dfrac{h^2}{2m}\left(\dfrac{3n}{8\pi}\right)^{2/3}$

$$E_F = \left[\frac{(6.625\times10^{-34}\text{ Js})^2}{(2\times9.11\times10^{-31}\text{ kg})}\right]\left[\frac{3\times2.54\times10^{28}\text{ electrons/m}^3}{8\pi}\right]^{2/3}$$

$$E_F = 5.04\times10^{-19}\text{ J} = 3.15\text{ eV}$$

(c) $v_F = \left(\dfrac{2E_F}{m}\right)^{1/2} = \left[\dfrac{2\times5.04\times10^{-19}\text{ J}}{9.11\times10^{-31}\text{ kg}}\right]^{1/2}$

$$v_F = 1.05\times10^{6}\text{ m/s}$$

10-23 $d = 1\text{ mm} = 10^{-3}\text{ m}$; $V = (10^{-3}\text{ m})^3 = 10^{-9}\text{ m}^3$

The <u>density of states</u> $= g(E) = CE^{1/2} = \left\{\dfrac{8(2)^{1/2}\pi m_e^{3/2}}{h^3}\right\}E^{1/2}$

$$g(E) = 8(2)^{1/2}\pi(9.11\times10^{-31}\text{ kg})^{3/2}\frac{[(4.0\text{ eV})(1.6\times10^{-19}\text{ J/eV})]^{1/2}}{(6.626\times10^{-34}\text{ Js})^3}$$

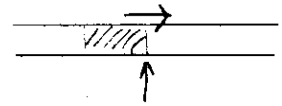
$$g(E) = (8.50\times10^{46})\text{m}^{-3}\text{ J}^{-1} = (1.36\times10^{28})\text{m}^{-3}\text{ eV}^{-1}$$

$$f_{FD}(E) = \frac{1}{e^{(E-E_F)/k_BT}+1} \text{ or}$$

$$f_{FD}(4.0\text{ eV}) = \frac{1}{e^{(4.0-5.5)/(8.6\times10^{-5}\text{ eV/K})(300\text{ K})}+1} = \frac{1}{e^{-59}+1} = 1$$

So the total number of electrons $= N = g(E)(\Delta E)Vf_{FD}(E)$ or
$N = (1.36\times10^{28}\text{ m}^{-3}\text{ eV}^{-1})(0.025\text{ eV})(10^{-9}\text{ m}^3)(1) = 3.40\times10^{17}$.

W-23) If n is the number of conduction electrons per cm^3 and the electrons move at velocity v_d, then the number of electrons that drift pass any point in time Δt is



$$\Delta n = n A v_d \Delta t$$

where A is the cross sectional area of the wire. Each electron has charge e, so the net charge is

$$\Delta Q = e\,\Delta n = enA v_d \Delta t$$

So

$$I = enA v_d \qquad v_d = I/enA$$

$I = 1A = 1C/s$

$A = (\pi/4) d^2 \qquad n = 5.9 \times 10^{28}/m^3 = 5.9 \times 10^{22}/cm^3$

$$v_d = \frac{1C/s}{(1.6\times10^{-19}C)(\pi/4)(0.02cm)^2(5.9\times10^{22}/cm^3)} = \boxed{0.34\ cm/s}$$

12-9 (a) $\int_0^\infty \left(\frac{N}{\tau}\right) e^{-t/\tau} dt = -Ne^{-t/\tau}\Big|_0^\infty = -N\left[e^{-\infty} - e^0\right] = N$

(b) $\bar{t} = \left(\frac{1}{N}\right)\int_0^\infty \left(\frac{tN}{\tau}\right) e^{-t/\tau} dt = \tau\int_0^\infty \left(\frac{t}{\tau}\right) e^{-t/\tau}\frac{dt}{\tau} = \tau\int_0^\infty z e^{-z} dz$

$$z = u \qquad dv = e^{-z}dz$$
$$dz = du \qquad v = -e^{-z}$$

so $\int_0^\infty z e^{-z} dz = \left(-ze^{-z}\right)\Big|_0^\infty + \int_0^\infty e^{-z} dz = 0 - e^{-z}\Big|_0^\infty = 1$. Therefore, $\bar{t} = \tau$.

(c) Similarly $\overline{t^2} = \left(\frac{1}{N}\right)\int_0^\infty \left(\frac{t^2 N}{\tau}\right) e^{-t/\tau} dt$. Integrating by parts twice, gives $\overline{t^2} = 2\tau^2$.

12-10 (a) $\tau = \frac{m_e}{ne^2\rho}$

n = number of electrons/m^3

$= (1\text{ electron/atom})(6.02\times10^{26}\text{ atoms/k mole})\times(10.5\times10^3\text{ kg/m}^3)(1\text{ k mole/108 kg})$

$= 5.85\times10^{28}\text{ electrons/m}^3$

so $\tau = \frac{9.11\times10^{-31}\text{ kg}}{(1.60\times10^{-8}\ \Omega\text{m})(5.85\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})^2} = 3.80\times10^{-14}\text{ s}.$

(b) As shown in Example 12.1, the rms speed of an electron at room temperature is about 1.2×10^{5} m/s. In 3.80×10^{-14} s the electron would travel a distance

$$L=v_{\text{rms}}\tau=\left(1.2\times10^{5}\ \text{m/s}\right)\times3.80\times10^{-14}\ \text{s}=4.6\times10^{-9}\ \text{m}=46\ \text{Å}$$

(c) 10–20 lattice spacings.

12-13 (a) We assume all expressions still hold with v_{rms} replaced by v_{F}.

$$\tau=\frac{\sigma m_e}{ne^2}$$

$$\sigma=\frac{1}{\rho}=\left(1.60\times10^{-8}\right)^{-1}(\Omega\ \text{m})^{-1}=6.25\times10^{7}\ (\Omega\ \text{m})^{-1}$$

$$n=\frac{\#\text{ of }e^-}{m^3}=\left(\frac{1\ e^-}{\text{atom}}\right)\left(6.02\times10^{26}\ \text{atoms/k mole}\right)\left(10.5\times10^{3}\ \text{kg/m}^3\right)\left(\frac{1\ \text{kmole}}{108\ \text{g}}\right)$$

$$n=5.85\times10^{28}\ e^-/\text{m}^3$$

so $\tau=\dfrac{\left(6.25\times10^{7}\right)(\Omega\ \text{m})^{-1}\left(9.11\times10^{-31}\ \text{kg}\right)}{\left(5.85\times10^{28}\ e^-/\text{m}^3\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}=3.80\times10^{-14}$ s (no change of course from Equation 12.10).

(b) Now $L=v_{\text{F}}\tau$ and $v_{\text{F}}=\left(\dfrac{2E_{\text{F}}}{m}\right)^{1/2}$

$$v_{\text{F}}=\left[\frac{2\times5.48\ \text{eV}\times1.6\times10^{-19}\ \text{J/eV}}{9.11\times10^{-31}\ \text{kg}}\right]^{1/2}=1.39\times10^{6}\ \text{m/s}.$$

$$L=\left(1.39\times10^{6}\ \text{m/s}\right)\left(3.8\times10^{-14}\ \text{s}\right)=5.27\times10^{-8}\ \text{m}=527\ \text{Å}=52.7\ \text{nm}$$

(c) The approximate lattice spacing in silver may be calculated from the density and the molar weight. The calculation is the same as the n calculation. Thus, $(\#\text{ of Ag atoms})/\text{m}^3=5.85\times10^{28}$. Assuming each silver atom fits in a cube of side, d,

$$d^3=\left(5.85\times10^{28}\right)^{-1}\ \text{m}^3/\text{atom}$$

$$d=2.57\times10^{-10}\ \text{m}$$

So $\dfrac{L}{d}=\dfrac{5.27\times10^{-8}}{2.57\times10^{-10}}=205$.