Homework 12 Solutions

13-10 (a) $R = R_0A^{1/3} = 1.2\times10^{-15}\ A^{1/3}$ m. When $A = 12$, $R = 2.75\times10^{-15}$ m.

(b) $F = \frac{k(Z-1)e^2}{R^2} = \frac{(9\times10^9\ \text{N m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2(Z-1)}{R^2}$. When $Z = 6$ and $R = 2.75\times10^{-15}$ m, $F = 152$ N.

(c) $U = \frac{kq_1q_2}{R} = \frac{k(Z-1)e^2}{R} = \frac{(9\times10^9\ \text{N m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2(Z-1)}{R}$. When $Z = 6$ and $R = 2.75\times10^{-15}$ m, $U = 4.19\times10^{-13}$ J $= 2.62$ MeV.

(d) $A = 238$; $Z = 92$, $R = 7.44\times10^{-15}$ m, $F = 379$ N and $U = 2.82\times10^{-12}$ J $= 17.6$ MeV

13-12 $E_b = \left[26m_{1\text{H}} + 30m_{\text{n}} - m(^{56}\text{Fe})\right](931.5\ \text{MeV/u}) = 492\ \text{MeV}$

$\frac{E_b}{A} = \frac{492}{56} = 8.79$ MeV/nucleon; agrees with Figure 13.10.

13-14 (a) For $^{15}_{8}\text{O}$ we have, using Equation 13.4

$$E_b = [(8)(1.007\,825) + 7(1.008\,665) - (15.003\,066)]\ \text{u}(931.5\ \text{MeV/u}) = 111.96\ \text{MeV}$$

For $^{15}_{7}\text{N}$ we have

$$E_b = [(7)(1.007\,825) + 8(1.008\,665) - (15.000\,109)]\ \text{u}(931.5\ \text{MeV/u}) = 115.49\ \text{MeV}$$

Therefore $\Delta E_b = 3.53$ MeV.

(b) Use Equation 13.4 to find for $^{23}_{11}\text{Na}$; $\frac{E_b}{A} = 8.11$ MeV/nucleon and for $^{23}_{12}\text{Mg}$; $\frac{E_b}{A} = 7.90$ MeV/nucleon. The binding energy per nucleon is greater for $^{23}_{11}\text{Na}$ by 0.210 MeV. In both cases, the isobars with more protons experience more coulomb repulsion and are less tightly bound.

13-21 (a) Write Equation 13.10 as $\frac{R}{R_0} = e^{-\lambda t}$ so that $\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right)$. In this case $\frac{R_0}{R} = 5$ when $t = 2$ h, so $\lambda = \frac{1}{2\ \text{h}}\ln 5 = 0.805\ \text{h}^{-1}$.

(b) $T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.805\ \text{h}^{-1}} = 0.861$ h

13-22 (a) $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{8.04\ \text{day}} = 9.98\times10^{-7}\ \text{s}^{-1}$

(b) $\left|\frac{dN}{dt}\right| = R = \lambda N$ so $N = \frac{R}{\lambda}$

$$N = \frac{(0.5\times10^{-6}\ \text{Ci})(3.7\times10^{10}\ \text{decays/s Ci})}{9.98\times10^{-7}/\text{s}} = 1.85\times10^{10}\ \text{nuclei}$$

13-24 From Equation 13.9, the fraction of nuclei remaining after five years will be

$$\frac{N}{N_0}=e^{-\lambda t}=e^{-(\ln 2/T_{1/2})t}=e^{-(0.693/12.33)(5)}=0.755.$$

Therefore the percentage decaying during the interval will be $(1-0.755)$ or 24.5%.

13-29 The number of nuclei that decay during the interval will be

$$N_1-N_2=N_0\left(e^{-\lambda t_1}-e^{-\lambda t_2}\right).$$

First we find λ;

$$\lambda=\frac{\ln 2}{T_{1/2}}=\frac{0.693}{64.8\text{ h}}=0.0107\text{ h}^{-1}=2.97\times10^{-6}\text{ s}^{-1}\text{ and}$$

$$N_0=\frac{R_0}{\lambda}=\frac{(40\ \mu\text{Ci})(3.7\times10^4\ \text{dps}/\mu\text{Ci})}{2.97\times10^{-6}\text{ s}^{-1}}=4.98\times10^{11}\text{ nuclei}$$

Using these values we find

$$N_1-N_2=\left(4.98\times10^{11}\right)\left[e^{-(0.0107\text{ h}^{-1})(10\text{ h})}-e^{-(0.0107\text{ h}^{-1})(12\text{ h})}\right].$$

Hence, the number of nuclei decaying during the interval is

$$N_1-N_2=9.46\times10^9\text{ nuclei}.$$

13-44 $Q=[m(R_n)-m(\text{He})-m(\text{Po})]931.5\text{ MeV}$
$Q=[220.011368-4.002603-216.001888]\text{ u}(931.5\text{ MeV/u})=6.40\text{ MeV}$
If we assume no recoil, $K_\alpha=Q=6.40\text{ MeV}$

13-45 $Q=(m_{\text{initial}}-m_{\text{final}})(931.5\text{ MeV/u})$

(a) $Q=m\left({}^{40}_{20}\text{Ca}\right)-m\left(e^+\right)-m\left({}^{40}_{19}\text{K}\right)=(39.96259\text{ u}-0.0005486\text{ u}-39.96400\text{ u})(931.5\text{ MeV/u})$
$=-1.82\text{ MeV}$
$Q<0$ so the reaction cannot occur.

(b) Using the handbook of Chemistry and Physics
$Q=m\left({}^{98}_{44}\text{Ru}\right)-m\left({}^{4}_{2}\text{He}\right)-m\left({}^{94}_{42}\text{Mo}\right)=(97.9055\text{ u}-4.0026\text{ u}-93.9047\text{ u})(931.5\text{ MeV/u})$
$=-1.68\text{ MeV}$
$Q<0$ so the reaction cannot occur.

(c) Using the handbook of *Chemistry and Physics*
$Q=m\left({}^{144}_{60}\text{Nd}\right)-m\left({}^{4}_{2}\text{He}\right)-m\left({}^{140}_{58}\text{Ce}\right)=(143.9099\text{ u}-4.0026\text{ u}-139.9054\text{ u})$
$\times(931.5\text{ MeV/u})=1.86\text{ MeV}$
$Q>0$ so the reaction can occur.

13-51 In the decay ${}^3_1\text{H} \to {}^3_2\text{He} + e + \bar{\nu}$ the energy released is: $E = (\Delta m)c^2 = [M_{1H} - M_{3He}]c^2$ since the mass of the antineutrino is negligible and the mass of the electron is accounted for in the atomic masses of ${}^3_1\text{H}$ and ${}^3_2\text{He}$. Thus,

$$E = (3.016\,049\text{ u} - 3.016\,029\text{ u})(931.5\text{ MeV/u}) = 0.018\,6\text{ MeV} = 18.6\text{ keV}.$$

13-55 (a) Starting with $N = 0$ radioactive atoms at $t = 0$, the rate of increase is (production-decay)

$$\frac{dN}{dt} = R - \lambda N$$
$$dN = (R - \lambda N)dt$$

Variables are separable

$$\int_{N=0}^{N} \frac{dN}{R - \lambda N} = \int_{t=0}^{t} dt - \left(\frac{1}{\lambda}\right)\ln\left(\frac{R - \lambda N}{R}\right) = t$$
$$\ln\left(\frac{R - \lambda N}{R}\right) = -\lambda t$$
$$\left(\frac{R - \lambda N}{R}\right) = e^{-\lambda t}$$
$$1 - \left(\frac{\lambda}{R}\right)N = e^{-\lambda t}$$
$$N = \left(\frac{R}{\lambda}\right)\left(1 - e^{-\lambda t}\right)$$

(b) $$\frac{dN}{dt} = R - \lambda N_{max}$$
$$N_{max} = \frac{R}{\lambda}$$