Homework 2 Solutions

1-20 $u = \dfrac{v+u'}{1+vu'/c^2} = \dfrac{0.90c+0.70c}{1+(0.90c)(0.70c)/c^2} = 0.98c$

1-23 (a) Let event 1 have coordinates $x_1 = y_1 = z_1 = t_1 = 0$ and event 2 have coordinates $x_2 = 100$ mm, $y_2 = z_2 = t_2 = 0$. In S', $x_1' = \gamma(x_1 - vt_1) = 0$, $y_1' = y_1 = 0$, $z_1' = z_1 = 0$, and $t_1' = \gamma\left[t_1 - \left(\frac{v}{c^2}\right)x_1\right] = 0$, with $\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-1/2}$ and so $\gamma = \left[1-(0.70)^2\right]^{-1/2} = 1.40$. In system S', $x_2' = \gamma(x_2 - vt_2) = 140$ m, $y_2' = z_2' = 0$, and

$$t_2' = \gamma\left[t_2 - \left(\frac{v}{c^2}\right)x_2\right] = \frac{(1.4)(-0.70)(100\text{ m})}{3.00\times10^8\text{ m/s}} = -0.33\ \mu\text{s}.$$

(b) $\Delta x' = x_2' - x_1' = 140$ m

(c) Events are not simultaneous in S', event 2 occurs 0.33 μs earlier than event 1.

1-29 (a) A spaceship, reference frame S', moves at speed v relative to the Earth, whose reference frame is S. The space ship then launches a shuttle craft with velocity v in the forward direction. The pilot of the shuttle craft then fires a probe with velocity v in the forward direction. Use the relativistic compounding of velocities as well as its inverse transformation: $u_x' = \dfrac{u_x - v}{1-\left(u_x v/c^2\right)}$, and its inverse $u_x = \dfrac{u_x' + v}{1+\left(u_x' v/c^2\right)}$. The above variables are defined as: v is the spaceship's velocity relative to S, u_x' is the velocity of the shuttle craft relative to S', and u_x is the velocity of the shuttle craft relative to S. Setting u_x' equal to v, we find the velocity of the shuttle craft relative to the Earth to be: $u_x = \dfrac{2v}{1+(v/c)^2}$.

(b) If we now take S to be the shuttle craft's frame of reference and S' to be that of the probe whose speed is v relative to the shuttle craft, then the speed of the probe relative to the spacecraft will be, $u_x' = \dfrac{2v}{1+(v/c)^2}$. Adding the speed relative to S yields:

$u_x = \left[\dfrac{3+(v/c)^2}{1+2(v/c)^2}\right] = \dfrac{3v+v^3/c^3}{1+2v^2/c^2}$. Using the Galilean transformation of velocities, we see that the spaceship's velocity relative to the Earth is v, the velocity of the shuttle craft relative to the space ship is v and therefore the velocity of the shuttle craft relative to the Earth must be $2v$ and finally the speed of the probe must be $3v$. In the limit of low $\left(\dfrac{v}{c}\right)^2$, u_x reduces to $3v$. On the other hand, using relativistic addition of velocities, we find that $u_x = c$ when $v \to c$.

1-36 Let Suzanne be fixed in reference from S and see the two light-emission events with coordinates $x_1 = 0$, $t_1 = 0$, $x_2 = 0$, $t_2 = 3\ \mu s$. Let Mark be fixed in reference frame S' and give the events coordinate $x_1' = 0$, $t_1' = 0$, $t_2' = 9\ \mu s$.

(a) Then we have

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) = 9\ \mu s = \frac{1}{\sqrt{1-v^2/c^2}}(3\ \mu s - 0) = \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{3} = \frac{v^2}{c^2} = \frac{8}{9} = v = 0.943c.$$

(b) $$x_2' = \gamma(x_2 - vt_2) = 3(0 - 0.943c \times 3 \times 10^{-6}\ \text{s})\left(\frac{3\times 10^8\ \text{m/s}}{c}\right) = 2.55\times 10^3\ \text{m}$$

1-38 Let the S be the Earth frame of reference. Then $v = -0.7c$. The components of the velocity of the first spacecraft are $u_x = (0.6c)\cos 50° = 0.386c$ and $u_y = (0.6c)\sin 50° = 0.459c$. As measured from the S' frame of the second spacecraft,

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.386c - (-0.7c)}{1-(0.386c)(-0.7c)/c^2} = \frac{1.086c}{1.27} = 0.855c$$

$$u_y' = \frac{u_y}{\gamma(1 - u_x v/c^2)} = \frac{0.459\sqrt{1-(0.7)^2}}{1-(0.386)(0.7)} = \frac{0.459c(0.714)}{1.27} = 0.258c$$

The magnitude of $\mathbf{u}'$ is $\sqrt{(0.855c)^2 + (0.285c)^2} = 0.893c$, and its direction is at $\tan^{-1}\frac{0.258c}{0.855c} = 16.8°$ above the x'-axis.

2-1 $$p = \frac{mv}{\left[1-(v^2/c^2)\right]^{1/2}}$$

(a) $$p = \frac{(1.67\times 10^{-27}\ \text{kg})(0.01c)}{\left[1-(0.01c/c)^2\right]^{1/2}} = 5.01\times 10^{-21}\ \text{kg}\cdot\text{m/s}$$

(b) $$p = \frac{(1.67\times 10^{-27}\ \text{kg})(0.5c)}{\left[1-(0.5c/c)^2\right]^{1/2}} = 2.89\times 10^{-19}\ \text{kg}\cdot\text{m/s}$$

(c) $$p = \frac{(1.67\times 10^{-27}\ \text{kg})(0.9c)}{\left[1-(0.9c/c)^2\right]^{1/2}} = 1.03\times 10^{-18}\ \text{kg}\cdot\text{m/s}$$

(d) $\frac{1.00\ \text{MeV}}{c} = \frac{1.602\times 10^{-13}\ \text{J}}{2.998\times 10^8\ \text{m/s}} = 5.34\times 10^{-22}\ \text{kg}\cdot\text{m/s}$ so for (a)

$$p = \frac{(5.01\times 10^{-21}\ \text{kg}\cdot\text{m/s})(100\ \text{MeV}/c)}{5.34\times 10^{-22}\ \text{kg}\cdot\text{m/s}} = 9.38\ \text{MeV}/c$$

Similarly, for (b) $p = 540\ \text{MeV}/c$ and for (c) $p = 1\,930\ \text{MeV}/c$.

2-8 (a) $E_R = mc^2 = (1.67\times10^{-27}\ \text{kg})(3\times10^8\ \text{m/s})^2 = 1.503\times10^{-10}\ \text{J} = 939.4\ \text{MeV}$ (Numerical round off gives a slightly larger value for the proton mass)

(b) $$E = \gamma mc^2 = \frac{1.503\times10^{-10}\ \text{J}}{\left(1-(0.95c/c)^2\right)^{1/2}} = 4.813\times10^{-10}\ \text{J} \approx 3.01\times10^3\ \text{MeV}$$

(c) $K = E - mc^2 = 4.813\times10^{-10}\ \text{J} - 1.503\times10^{-10}\ \text{J} = 3.31\times10^{-10}\ \text{J} = 2.07\times10^3\ \text{MeV}$

2-15 (a) $K = \gamma mc^2 - mc^2 = Vq$ and so, $\gamma^2 = \left(1+\frac{Vq}{mc^2}\right)^2$ and $\frac{v}{c} = \left\{1-\left(1+\frac{Vq}{mc^2}\right)^{-2}\right\}^{1/2}$

$$\frac{v}{c} = \left\{1 - \frac{1}{1+\left(5.0\times10^4\ \text{eV}/0.511\ \text{MeV}\right)^2}\right\}^{1/2} = 0.4127$$

or $v = 0.413c$.

(b) $K = \frac{1}{2}mv^2 = Vq$

$$v = \left(\frac{2Vq}{m}\right)^{1/2} = \left\{\frac{2\left(5.0\times10^4\ \text{eV}\right)}{0.511\ \text{MeV}/c^2}\right\}^{1/2} = 0.442c$$

(c) The error in using the classical expression is approximately $\frac{3}{40}\times100\%$ or about 7.5% in speed.

2-20 $\Delta m = m\ \ - m_p - m_e = 1.008\,665\ \text{u} - 1.007\,276\ \text{u} - 0.000\,548\,5\ \text{u} = 8.404\times10^{-4}\ \text{u}$
$E = c^2\left(8.404\times10^{-4}\ \text{u}\right) = \left(8.404\times10^{-4}\ \text{u}\right)(931.5\ \text{MeV/u}) = 0.783\ \text{MeV}$.

2-33 The energy that arrives in one year is

$$E = \mathcal{P}\,\Delta t = \left(1.79\times10^{17}\ \text{J/s}\right)\left(3.16\times10^7\ \text{s}\right) = 5.66\times10^{24}\ \text{J}.$$

Thus, $m = \frac{E}{c^2} = \frac{5.66\times10^{24}\ \text{J}}{\left(3.00\times10^8\ \text{m/s}\right)^2} = 6.28\times10^7\ \text{kg}$.