Homework 5 Solutions

W4) (a) The distance of closest approach is given by

$$r_{min} = \frac{2Ze^2}{4\pi\epsilon_0}\frac{1}{E_\alpha} = \frac{(2)(29)(1.44\,\text{eV}\cdot\text{nm})}{6\ \text{MeV}} = 1.39\times10^{-5}\ \text{nm}$$

$$\boxed{r_{min} = 13.9\ \text{fm}}$$

(b) To get $r_{min} = 5\,\text{fm}$ we would need

$$E_\alpha = \frac{2Ze^2}{4\pi\epsilon_0}\cdot\frac{1}{r_{min}} = \frac{(2)(29)(1.44\,\text{eV}\cdot\text{nm})}{5\times10^{-6}\ \text{nm}} \Rightarrow \boxed{E_\alpha = 16.7\ \text{MeV}}$$

4-8 (a) From Equation 4.16 we have $\Delta n \propto \left(\frac{\sin\phi}{2}\right)^{-4}$ or $\Delta n_2 = \Delta n_1 \frac{\left(\frac{\sin\phi_1}{2}\right)^4}{\left(\frac{\sin\phi_2}{2}\right)^4}$. Thus the number of α's scattered at 40 degrees is given by

$$\Delta n_2 = (100\ \text{cpm})\frac{\left(\sin\frac{20}{2}\right)^4}{\left(\sin\frac{40}{2}\right)^4} = (100\ \text{cpm})\left(\frac{\sin 10}{\sin 20}\right)^4 = 6.64\ \text{cpm}.$$

Similarly

Δn at 60 degrees = 1.45 cpm
Δn at 80 degrees = 0.533 cpm
Δn at 100 degrees = 0.264 cpm

(b) From 4.16 doubling $\left(\frac{1}{2}\right)m_\alpha v_\alpha^2$ reduces Δn by a factor of 4. Thus Δn at 20 degrees $= \left(\frac{1}{4}\right)(100\ \text{cpm}) = 25$ cpm.

(c) From 4.16 we find $\frac{\Delta n_{Cu}}{\Delta n_{Au}} = \frac{Z_{Cu}^2 N_{Cu}}{Z_{Au}^2 N_{Au}}$, $Z_{Cu} = 29$, $Z_{Au} = 79$.

$$\begin{aligned} N_{Cu} &= \text{number of Cu nuclei per unit area} \\ &= \text{number of Cu nuclei per unit volume * foil thickness} \\ &= \left[(8.9\ \text{g/cm}^3)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{63.54\ \text{g}}\right)\right]t = 8.43\times10^{22}t \end{aligned}$$

$$N_{Au} = \left[(19.3\ \text{g/cm}^3)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{197.0\ \text{g}}\right)\right]t = 5.90\times10^{22}t$$

So $\Delta n_{Cu} = \Delta n_{Au}(29)^2\frac{8.43\times10^{22}}{(79)^2}(5.90\times10^2) = (100)\left(\frac{29}{79}\right)^2\left(\frac{8.43}{5.90}\right) = 19.3$ cpm.

W.5) (a) The radius of the orbit is given by

$$r_n = \left[\frac{4\pi\epsilon_0}{e^2}\right]\frac{\hbar^2}{m_e}\cdot n^2 = \left(\frac{4\pi\epsilon_0}{e^2}\right)\frac{(hc/2\pi)^2}{m_e c^2}\ n^2$$

$$= \left(\frac{1}{1.44\,eV\cdot nm}\right)(1240\,eV\cdot nm)^2\left(\frac{1}{2\pi}\right)^2\frac{1}{5.11\times10^5\,eV}\cdot(2)^2$$

$$\boxed{r_n = 0.2118\ nm}$$

(b) We can find the velocity using Bohr's quantization condition:

$$mvr = n\hbar$$

so

$$v = n\hbar/mr = n\left[(hc/2\pi)/(mc^2)(r)\right]c$$

$$= (2)\left[\frac{1240\,eV\cdot nm}{(2\pi)}\ \frac{1}{5.11\times10^5\,eV}\cdot\frac{1}{0.2118\,nm}\right]c \Rightarrow \boxed{v = 0.00365\,c}$$

(c) The kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(mc^2)\left(\frac{v}{c}\right)^2 = \frac{1}{2}(5.11\times10^5\,eV)\left(\frac{1.094\times10^6\,m/s}{3\times10^8\,m/s}\right)^2$$

$$\Rightarrow \boxed{K = 3.401\,eV}$$

(d) The potential energy is

$$V = -\frac{e^2}{4\pi\epsilon_0}\cdot\frac{1}{r} = -(1.44\,eV\cdot nm)\left(\frac{1}{.2118\,nm}\right) \Rightarrow \boxed{V = -6.801\,eV}$$

4-25 **(a)** $\Delta E = hf = (13.6\text{ eV})\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$ or $f = (13.6\text{ eV})\left(\frac{\frac{1}{9}-\frac{1}{16}}{4.14\times10^{-15}\text{ eV s}}\right) = 1.60\times10^{14}\text{ Hz}$

(b) $T = \frac{2\pi r_n}{v}$ so $f_{rev} = \frac{1}{T} = \frac{v}{2\pi r_n}$. Using $v = \left(\frac{ke^2}{m_e r_n}\right)^{1/2}$, $f_{rev} = \left(\frac{ke^2}{mr_n}\right)^{1/2}$. For $n = 3$, $r_3 = (3)^2 a_0$ and

$$f_{rev} = \frac{(8.99\times10^9\text{ Nm}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{\frac{[(9.11\times10^{-31}\text{ kg})(9)(5.29\times10^{-11}\text{ m})]^{1/2}}{(2)(3.14)(9)(5.29\times10^{-11}\text{ m})}}$$

$$f_{rev} = 2.44\times10^{14}\text{ Hz } (n=3)$$

$$f_{rev} = 1.03\times10^{14}\text{ Hz } (n=4)$$

Thus the photon frequency is about halfway between the two frequencies of the revolution.

W6) If the electron in its orbit has energy $E = K + V$, then the ionization energy is just $I = -E$ (E is negative and the ionization energy is positive).

(a) For ordinary hydrogen

$$E = -\frac{1}{2}\left[\frac{e^2}{4\pi\epsilon_0}\cdot\frac{1}{\hbar c}\right]^2 mc^2 \cdot \frac{1}{n^2}$$

Thus for $n = 3$ we get

$$I = \frac{1}{2}\left[\frac{1}{137}\right]^2 (5.11\times10^5\,eV)\left(\frac{1}{9}\right) \Rightarrow \boxed{I = 1.513\,eV}$$

(b) For He^+ the nucleus has charge $2e$, and to get the appropriate formula for E we replace e^2 by $Ze^2 = 2e^2$ ⇒ we have

$$I = \frac{1}{2}\left[\frac{2}{137}\right]^2 (5.11\times10^5\,eV)\left(\frac{1}{4}\right) \Rightarrow \boxed{I = 13.6\,eV}$$

W7) For $\lambda = 59.0$ nm the photons have energy

$$E = hf = hc/\lambda = (1240\,eV\cdot nm)/(59.0\,nm) = 21.02\,eV$$

In the ground state of hydrogen the electron has energy $E = -13.6$ eV, so if a photon of energy $E = 21.02$ eV is absorbed, the emitted electron will have kinetic energy

$$K = 21.02\,eV - 13.6\,eV \Rightarrow \boxed{K = 7.42\,eV}$$

W8) The formula for the wavelengths is given by equation (4.33). Note that in this formula

$$k = \frac{1}{4\pi\epsilon_0}$$

and

$$a_0 = \left(\frac{\hbar^2}{m}\right)\frac{4\pi\epsilon_0}{e^2}$$

so we have

$$\frac{1}{\lambda} = \frac{1}{2}\frac{1}{hc}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{m}{\hbar^2}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$= \frac{1}{2}\frac{1}{hc}\left[\frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c}\right]^2 (mc^2)\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

For the first Balmer line we want $n_f = 2$ and $n_i = 3$ so

$$\frac{1}{\lambda} = \frac{1}{2}\left(\frac{1}{1240\,eV\cdot nm}\right)\left[\frac{1}{137}\right]^2\left[\frac{1}{4}-\frac{1}{9}\right] mc^2 = \left[2.984\times10^{-9}/eV\cdot nm\right] mc^2$$

$\Rightarrow$

$$\lambda = 3.351\times10^{8}\ eV\cdot nm\ /\ mc^2$$

Now for m we need to put in the reduced mass $m = \frac{m_e M}{m_e + M}$ $\Rightarrow$
$mc^2 = m_e c^2\left[M/(M+m)\right]$

For hydrogen $mc^2 = (5.11\times10^5 eV)\left[\frac{1.0078}{1.0078u + 5.486\times10^{-4}u}\right]$
$= 510,722\ eV$

$\Rightarrow$ $\lambda = 656.207\ nm$

For deuterium

$mc^2 = (5.11\times10^5 eV)\left[\frac{2.0141u}{2.0141u + 5.486\times10^{-4}u}\right]$
$= 510,861\ eV$

$\lambda = 656.028\ nm$

$\Rightarrow$ $\Delta\lambda = 656.207\ nm - 656.028\ nm$

$$\boxed{\Delta\lambda = 0.179\ nm}$$

W9) For the energy of the lowest state $(n=1)$ we use the Bohr model result

$$E = -\frac{1}{2}\left[\frac{e^2}{4\pi\epsilon_0}\right]^2\frac{m}{\hbar^2} = -\frac{1}{2}\left[\frac{1}{4\pi\epsilon_0}\cdot\frac{e^2}{\hbar c}\right]^2 mc^2$$

For m we use the reduced mass for the muon-proton atom

$$\mu = \frac{m_\mu m_p}{m_\mu + m_p} = \frac{(105.7\ MeV/c^2)(938.28\ MeV/c^2)}{(105.7\ MeV/c^2)+(938.28\ MeV/c^2)} = 94.998\ MeV/c^2$$

So

$$E = -\frac{1}{2}\left(\frac{1}{137}\right)^2(94.998\ MeV) \Rightarrow \boxed{E = -2531\ eV}$$

The radius of the orbit is

$$r = \left[\frac{4\pi\epsilon_0}{e^2}\right]\frac{(\hbar c)^2}{mc^2} = \left[\frac{1}{1.44\,\text{eV}\cdot\text{nm}}\right]\frac{[(1240\,\text{eV}\cdot\text{nm})/2\pi]^2}{94.998\,\text{MeV}} \Rightarrow \boxed{r = 2.85\times10^{-4}\,\text{nm}}$$

For the $n=2 \rightarrow n=1$ transition, the photon will have energy

$$E_\gamma = E_2 - E_1 = E_1\left[\tfrac{1}{4}-1\right] = -\tfrac{3}{4}E_1 = \tfrac{3}{4}\left[2531\,\text{eV}\right] = 1898\,\text{eV}$$

Thus the wavelength is

$$\lambda = hc/E = (1240\,\text{eV}\cdot\text{nm})/(1898\,\text{eV}) \Rightarrow \boxed{\lambda = 0.653\,\text{nm}}$$