Homework 6 Solutions

5-1 $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{Js}}{1.67\times10^{-27}\ \text{kg}}\left(10^6\ \text{m/s}\right) = 3.97\times10^{-13}\ \text{m}$

5-6 From Problem 5-2, a 50 keV electron has $\lambda = 5.36\times10^{-3}$ nm. A 50 keV proton has $K = 50\ \text{keV} << 2mc^2 = 1\,877\ \text{MeV}$ so we use $p = (2mK)^{1/2}$:

$$\lambda = \frac{h}{p} = \frac{h}{\left[(2)\left(\frac{938.3\ \text{MeV}}{c^2}\right)(50\ \text{keV})\right]^{1/2}} = \frac{hc}{[(2)(938.3\ \text{MeV})(50\ \text{keV})]^{1/2}}$$

$$= \frac{1\,240\ \text{eV nm}}{\left[(2)(938.3\times10^3\ \text{eV})(50\times10^3\ \text{eV})\right]^{1/2}} = 1.28\times10^{-4}\ \text{nm}$$

5-10 As $\lambda = 2a_0 = 2(0.052\,9)\ \text{nm} = 0.105\,8\ \text{nm}$ the energy of the electron is nonrelativistic, so we can use

$$p = \frac{h}{\lambda}\ \text{with}\ K = \frac{p^2}{2m};$$

$$K = \frac{h^2}{2m\lambda^2} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.058\times10^{-10}\ \text{m}\right)^2} = 21.5\times10^{-18}\ \text{J} = 134\ \text{eV}$$

This is about ten times as large as the ground-state energy of hydrogen, which is 13.6 eV.

5-15 For a free, non-relativistic electron $E = \frac{m_e v_0^2}{2} = \frac{p^2}{2m_e}$. As the wavenumber and angular frequency of the electron's de Broglie wave are given by $p = \hbar k$ and $E = \hbar\omega$, substituting these results gives the dispersion relation $\omega = \frac{\hbar k^2}{2m_e}$. So $v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m_e} = \frac{p}{m_e} = v_0$.

5-24 (a) $\Delta x \Delta p = \hbar$ so if $\Delta x = r$, $\Delta p \approx \frac{\hbar}{r}$

(b)

$$K = \frac{p^2}{2m_e} \approx \frac{(\Delta p)^2}{2m_e} = \frac{\hbar^2}{2m_e r^2}$$

$$U = -\frac{ke^2}{r}$$

$$E = \frac{\hbar^2}{2m_e r^2} - \frac{ke^2}{r}$$

(c) To minimize E take $\frac{dE}{dr} = -\frac{\hbar^2}{m_e r^3} + \frac{ke^2}{r^2} = 0 \Rightarrow r = \frac{\hbar^2}{m_e ke^2}$ = Bohr radius = a_0. Then

$$E = \left(\frac{\hbar}{2m_e}\right)\left(\frac{m_e ke^2}{\hbar^2}\right)^2 - ke^2\left(\frac{m_e ke^2}{\hbar^2}\right) = \frac{m_e k^2 e^4}{2\hbar^2} = -13.6\ \text{eV}.$$

5-28 (a) $\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(1.67\times10^{-27}\text{ kg})(0.4\text{ m/s})} = 9.93\times10^{-7}\text{ m}$

(b) $\sin\theta = \frac{\lambda}{2D} = \frac{9.93\times10^{-7}\text{ m}}{2(1.0\times10^{-3}\text{ m})} = 4.96\times10^{-4}$

As $\theta \approx \sin\theta$, $y \approx R\Theta = (10\text{ m})(4.96\times10^{-4}) = 4.96\text{ mm}$.

(c) We cannot say the neutron passed through one slit. We can only say it passed through the slits.

5-30 (a) From $E = \gamma m_e c^2$

$$\gamma = \frac{20\times10^3\text{ MeV}}{0.511\text{ MeV}} = 39\,139$$

$$p = \frac{E}{c}\left(\text{for } m_e c^2 << pc\right)$$

$$p = \frac{(2\times10^4\text{ MeV})(1.6\times10^{-13}\text{ J/MeV})}{3\times10^8\text{ m/s}} = 1.07\times10^{-17}\text{ kg}\cdot\text{m/s}$$

(b) $\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{1.07\times10^{-17}\text{ kg}\cdot\text{m/s}} \cong 6.2\times10^{-17}\text{ m}$. As the size of a nucleus is on the order of 10^{-14} m, the 20 GeV electrons would have a wavelength much smaller than the nucleus and allow details of the nuclear charge distribution to be revealed.

5-33 From the uncertainty principle, $\Delta E \Delta t \sim \hbar$ $\Delta mc^2 \Delta t = \hbar$. Therefore,

$$\frac{\Delta m}{m} = \frac{h}{2\pi c^2 \Delta t m} = \frac{h}{2\pi \Delta t E_{rest}} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{2\pi(8.7\times10^{-17}\text{ s})(135\times10^6\text{ eV})(1.6\times10^{-19}\text{ J/eV})} = 5.62\times10^{-8}.$$