

October 21, 2002

Physics 201

**EXAM 2**

**Print your name and section clearly on all five pages.** (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided.** Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points.** In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

*(Do not write below)***SCORE:**

Problem 1: \_\_\_\_\_

Problem 2: \_\_\_\_\_

Problem 3: \_\_\_\_\_

Problem 4: \_\_\_\_\_

**TOTAL:** \_\_\_\_\_

<b>Solution Key</b>
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Possibly useful information:

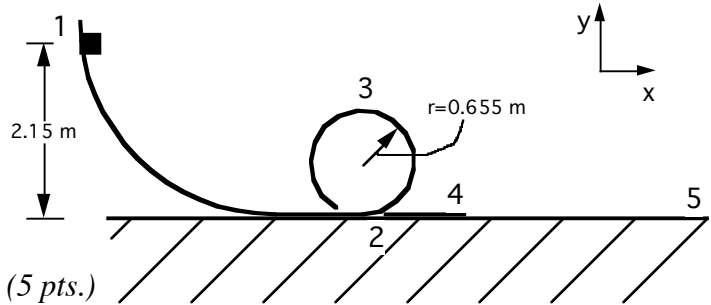
Acceleration due to gravity at the earth's surface:  $g = 9.80 \text{ m/s}^2$ 

$$\text{For } ax^2 + bx + c = 0, x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Moment of Inertia of a Solid Disc: } I = \frac{1}{2} MR^2$$

**PROBLEM 1 Loop the loop**

A block of mass 0.454 kg is released from rest at point 1 on the frictionless track of negligible thickness shown at right. Point 2 is just after the beginning of the loop, and point 3 is the top of the loop. The loop radius is  $r=0.655$  m.



a. What is the speed of the block at point 2? (5 pts.)

$$mgh = \frac{1}{2}mv^2 \quad v^2 = 2gh$$

$$v = \sqrt{2gh} = \sqrt{2(9.80 \text{ m/s}^2)(2.15 \text{ m})} = 6.49 \text{ m/s}$$

6.49 m/s

b. What is the normal force vector on the block at point 2? (5 pts.)

$$\sum F_{\text{radial}} = \frac{mv^2}{r} \quad F_{\text{norm}} - mg = \frac{mv^2}{r}$$

$$F_{\text{norm}} = m\left(g + \frac{v^2}{r}\right) = 0.454 \text{ kg} \left(9.80 \text{ m/s}^2 + \frac{(6.49 \text{ m/s})^2}{(0.655 \text{ m})}\right) = 33.7 \text{ N}$$

33.7 N  $\hat{y}$

c. What is the minimum speed the block must have at point 3, if it is to remain in contact with the track? (5 pts)

$$\frac{mv^2}{r} \geq mg \quad v \geq \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(0.655 \text{ m})} = 2.53 \text{ m/s}$$

2.53 m/s

d. The block completes the loop and slides off the end of the track (point 4). It then slides along the table until it comes to rest at point 5. The coefficient of kinetic friction between the block and the table is  $\mu_k = 0.530$ . What is the distance between point 4 and point 5? (5 pts)

$$\sum F_y = N - mg = 0 \quad N = mg$$

$$\sum F_x = F_{\text{fric}} = -\mu_k N = -\mu_k mg = ma_x \quad a_x = -\frac{\mu_k mg}{m} = -\mu_k g = -(0.530)(9.80 \text{ m/s}^2) = -5.19 \text{ m/s}^2$$

$$d = \frac{v_f^2 - v_i^2}{2a} = \frac{0 - (6.49 \text{ m/s})^2}{2(-5.19 \text{ m/s}^2)} = 4.06 \text{ m}$$

4.06 m

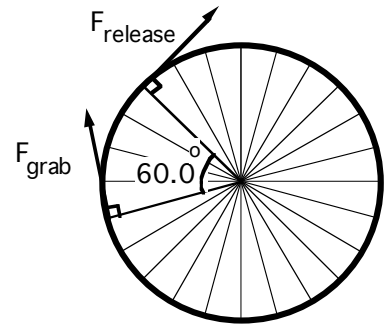
e. What is the total work done by friction on the block while it is sliding from point 4 to point 5? (5 pts.)

$$W_{\text{fric}} = \Delta K = \frac{1}{2}mv^2 = \frac{1}{2}(0.454 \text{ kg})(6.49 \text{ m/s})^2 = 9.57 \text{ J}$$

9.57 J

**PROBLEM 2 Wheel of Fortune**

Larry, Moe, and Curly are contestants on the game show “Wheel of Fortune,” which features a uniform 97.3 kg disc of radius 1.24 m. Curly grabs the rim of disc, which is at rest, and spins it. He applies a force of constant magnitude 221 N, always tangential to the disc rim, while the disc turns by  $\Delta\theta = 60.0^\circ$ , and then releases the disc. The diagram shows the force at the instant Curly grabs the disc, and the instant he releases it. (Note:  $I_{\text{disk}} = 1/2 MR^2$ )



a. What is the numerical value of the moment of inertia of the disc? (5 pts.)

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(97.3 \text{ kg})(1.24 \text{ m})^2 = 74.8 \text{ kg m}^2$$

74.8 kg m<sup>2</sup>

b. What is the total work Curly did on the disc? (5 pts.)

$$d = r\Delta\theta = (1.24 \text{ m})(60^\circ)\left(\frac{\pi}{180^\circ}\right) = 1.30 \text{ m}$$

$$W = \vec{F}_{\text{curly}} \cdot \vec{d} = F_{\text{curly}} d = (221\text{N})(1.30\text{m}) = 287 \text{ J}$$

287 J

c. What is the maximum angular speed of the disc? (5 pts.)

$$K_R = \frac{1}{2}I\omega^2 = W$$

$$\omega = \sqrt{\frac{2W}{I}} = \sqrt{\frac{2(287\text{J})}{74.8 \text{ kg m}^2}} = 2.77 \text{ rad/sec}$$

2.77 rad/s

d. What is the maximum instantaneous power Curly exerted on the disc? (5 pts.)

$$P_{\text{max}} = (\vec{F} \cdot \vec{v})_{\text{max}} = F v_{\text{max}} \quad v_{\text{max}} = \omega_{\text{max}} R = (2.77 \text{ rad/sec})(1.24 \text{ m}) = 3.43 \text{ m/s}$$

$$P_{\text{max}} = (221 \text{ N})(3.43 \text{ m/s}) = 759 \text{ W}$$

759 W

e. Larry accidentally leans against the disc, exerting a constant force tangential to the rim, and brings it to rest in 2.51 seconds. What is the torque Larry applied to the disc? (5 pts.)

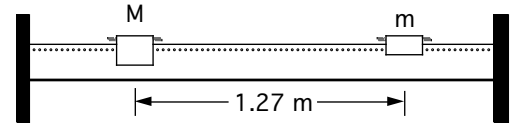
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 2.77 \text{ rad/s}}{2.51 \text{ s}} = -1.10 \text{ rad/s}^2$$

$$\tau = I\alpha = (74.8 \text{ kg m}^2)(1.10 \text{ rad/s}^2) = 82.6 \text{ Nm}$$

82.6 Nm

### PROBLEM 3 Colliding Gliders

Two gliders are sitting at rest 1.27 m apart on a frictionless air track, a large glider ( $M = 3.33 \text{ kg}$ ) and a small glider ( $m = 0.878 \text{ kg}$ ), as shown in the diagram. Each glider has perfectly elastic springs on both ends.



- a. Find the distance between the large glider and the center of mass of the two-glider system (5 pts).

$$r_{c.m.} = \frac{\sum_i m_i r_i}{\sum_i m_i} = \frac{(3.33 \text{ kg})(0.0 \text{ m}) + (0.878 \text{ kg})(1.27 \text{ m})}{3.33 \text{ kg} + 0.878 \text{ kg}} = 0.265 \text{ m}$$

0.265 m

- b. The small glider is given a push. It moves to the left and collides with the large glider. After the collision, the large glider moves to the left with speed  $v = 1.02 \text{ m/s}$ , and the small glider moves to the right with speed  $v = 1.42 \text{ m/s}$ . What was the speed of the small glider before the collision? (5 pts)

$$P_{\text{final}} = P_{\text{init}} \quad Mv_{Mf} + mv_{mf} = mv_{mi}$$

$$v_{mi} = v_{mf} + \frac{M}{m} v_{Mf} = 1.42 \text{ m/s} + \frac{3.33 \text{ kg}}{0.878 \text{ kg}} (-1.02 \text{ m/s}) = 2.45 \text{ m/s}$$

2.45 m/s

- c. During the collision, the two gliders were in contact for 0.115 seconds. What was the average force one glider exerted on the other during this time? (5 pts)

$$I = F_{\text{av}} \Delta t = \Delta p$$

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{(3.33 \text{ kg})(-1.02 \text{ m/s}) - 0 \text{ kgm/s}}{0.115 \text{ s}} = 29.5 \text{ N}$$

29.5 N

- d. After the collision, the gliders reach opposite ends of the track, bounce back, and approach each other once more. They collide for a second time. What is the velocity of the large glider after this second collision? (5 pts)

The velocities in the second collision are the reverse of the velocities in the first.

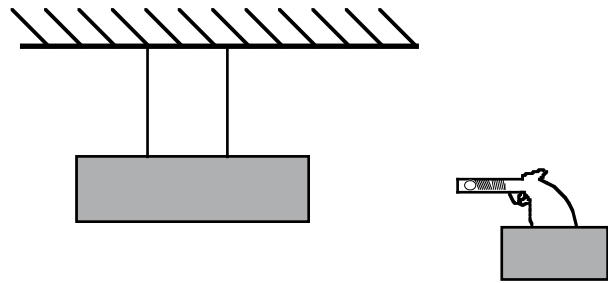
0 m/s

- e. What is the change in total mechanical energy from just before the first collision to just after the second collision? (5 pts)

0 J

**PROBLEM 4 Ballistic Pendulum**

A spring gun (spring constant 18,600 N/m) fires a projectile of mass 0.250 kg into a block of mass 10.2 kg which hangs from strings of length 1.89 m. When loading the gun the spring, initially at its uncompressed length, is compressed by 5.08 cm. The gun is fixed to a massive object so that it doesn't recoil.



a. What is the force needed to compress the spring by 5.08 cm? (5 pts.)

$$F = k \Delta x = (18,600 \text{ N/m})(0.0508 \text{ m}) = 945 \text{ N}$$

945 N

b. With what speed does the projectile leave the gun? (5 pts.)

$$W = K \quad \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$$

$$v = \Delta x \sqrt{\frac{k}{m}} = (0.0508 \text{ m}) \sqrt{\frac{18,600 \text{ N/m}}{0.25 \text{ kg}}} = 13.9 \text{ m/s}$$

13.9 m/s

c. The projectile embeds itself in the block. What is the resulting momentum of the projectile—block combination just after the collision? (5 pts.)

$$P_{\text{final}} = P_{\text{init}} = m_{\text{proj}} v_{\text{proj}} = (0.25 \text{ kg})(13.9 \text{ m/s}) = 3.46 \text{ kg m/s}$$

3.46 kg m/s

d. How much kinetic energy was lost in the collision? (5 pts.)

$$\Delta K = K_{\text{init}} - K_{\text{final}} \quad K_{\text{init}} = \frac{1}{2}m_{\text{proj}}v_{\text{proj}}^2 = \frac{1}{2}(0.250 \text{ kg})(13.9 \text{ m/s})^2 = 24.0 \text{ J}$$

$$K_{\text{final}} = \frac{1}{2}(m_{\text{block}} + m_{\text{proj}})v_{\text{final}}^2 \quad v_{\text{final}} = \frac{P_{\text{final}}}{m_{\text{block}} + m_{\text{proj}}} = \frac{3.46 \text{ kg m/s}}{10.2 \text{ kg} + 0.25 \text{ kg}} = 0.331 \text{ m/s}$$

$$K_{\text{final}} = \frac{1}{2}(10.2 \text{ kg} + 0.250 \text{ kg})(0.331 \text{ m/s})^2 = 0.573 \text{ J} \quad \Delta K = 24.0 \text{ J} - 0.573 \text{ J} = 23.4 \text{ J}$$

23.4 J

e. What is the maximum height of the block above its original position? (5 pts.)

$$K = U_g = mgh \quad h = \frac{K}{mg}$$

$$h = \frac{0.573 \text{ J}}{(10.45 \text{ kg})(9.80 \text{ m/s}^2)} = 0.00561 \text{ m} = 5.61 \text{ mm}$$

5.61 mm