September 24, 2003 Physics 201

EXAM 1

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

SOLUTION KEY

TOTAL: __________

Possibly useful information:

Acceleration due to gravity at the earth's surface: $g = 9.80\ \text{m/s}^2$

For $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

PROBLEM 1

A cannon at sea level protecting the entrance of a harbor has a muzzle velocity of 82.0 m/s.

a. What is the smallest angle *in degrees* with respect to the horizontal that will allow it to strike a ship 560.0 m away? *(5 pts.)*

$$R = \frac{V_0^2}{g} \sin 2\theta_0 \Rightarrow \theta_0 = \frac{1}{2}\sin^{-1}\left(\frac{gR}{V_0^2}\right) = \frac{1}{2}\sin^{-1}\left(\frac{(9.80\text{ms}^{-1})\cdot(560\text{m})}{(82.0\text{ms}^{-1})^2}\right) = \frac{\sin^{-1}(0.816)}{2} = 27.4° \text{ or } \cancel{62.7}°$$

27.4°

b. How many seconds after firing at this angle does the cannon ball strike the ship? *(5 pts.)*

$\rightarrow x - x_0 = (v_0\cos\theta_0)t \Rightarrow t = (x - x_0)/(v_0\cos\theta_0) = (560.0\text{ m})/[82.0\text{ ms}^{-1})(\cos 27.4°) =$

7.69 sec

c. What is the minimum distance away from the cannon for the ship to be out of its range? *(5 pts)*

$$R = \frac{V_0^2 \sin(2\cdot 45°)}{g} = \frac{V_0^2}{g} = \frac{(82.0\text{ms}^{-1})^2}{9.8\text{ms}^{-2}} =$$

686 m

d. A second cannon is now placed 30.0 m above the first and aimed at 45° from horizontal. What is the time of flight of a cannon ball from this second cannon? *(5 pts.)*

$$y = y_0 + V_{0y}t - \tfrac{1}{2}gt^2 = 0 \Rightarrow \tfrac{1}{2}gt^2 - V_0\sin(45°)t - y_0 = 0 \Rightarrow t = \frac{V_0\sin(45°) \pm \sqrt{(V_0\sin(45°))^2 - 4(\tfrac{1}{2}g)(-y_0)}}{2\cdot\tfrac{1}{2}g}$$

$$= \frac{58.0\text{ms}^{-1} \pm \sqrt{(58.0\text{ms}^{-1})^2 - 4(4.90\text{ms}^{-2})(-30.0\text{m})}}{9.8\text{ms}^{-2}}$$

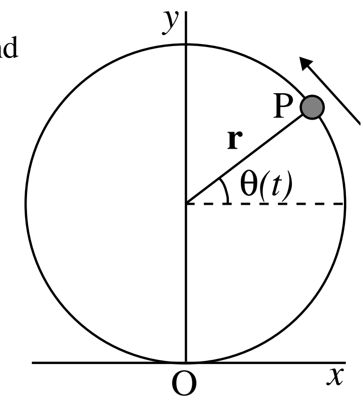
12.3 sec

e. At what distance from the first cannon does the second cannon's ball strike? *(5 pts.)*

$R' = V_{0x}t = (82.0\text{ ms}^{-1})(\cos 45°)(12.3\text{ s}) =$

715 m

PROBLEM 2



A particle P travels with constant speed on a circle of radius r = 3.00 m and completes one revolution in 20.0 s. The particle passes through O at t = 0.

a. Find the magnitude of the displacement vector of P with respect to O at t = 7.50 s. *(5 pts.)*

$$x = (3.00\,\text{m})\,sin\left(\frac{2\pi\cdot 7.50\text{s}}{20.0\text{s}}\right) = 2.12\,\text{m}$$

$$y = 3.00\,\text{m} + (3.00\,\text{m})\,cos\left(\frac{2\pi\cdot 7.50\text{s}}{20.0\text{s}}\right) = 5.12\,\text{m}$$

$$d = \sqrt{(2.12\text{m})^2 + (5.12\text{m})^2} =$$

5.54 m

b. Find the angle *in degrees* with the x axis of the vector from O to P at t = 7.50 s. *(5 pts.)*

$\phi = \tan^{-1}(y/x) = \tan^{-1}(5.12\text{ m}/2.12\text{ m}) =$

67.5°

c. During a 5.00 s interval what is the magnitude of the displacement of P? *(5 pts.)*

$$\frac{1}{4}\text{revolution}:\quad s = \sqrt{(3.00\text{ m})^2 + (3.00\text{ m})^2} =$$

4.24 m

d. What is the magnitude of the velocity of P? *(5 pts.)*

$v = 2\pi r/T = 2\pi(3.00\text{ m})/(20.0\text{ s}) =$

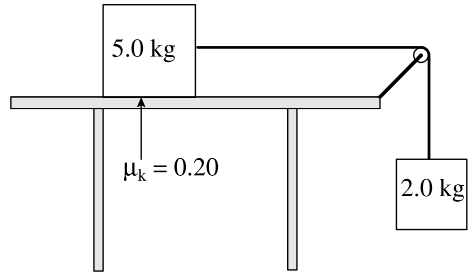
0.942 ms^{-1}

e. What is the magnitude of the acceleration of P? *(5 pts.)*

$a = v^2/r = (0.942\text{ms}^{-1})^2/(3.00\text{ m}) =$

0.296 ms^{-2}

PROBLEM 3



Two objects are connected by a rope running over a pulley as shown. The coefficient of kinetic friction between the 5.0 kg object and the table is 0.20. Ignore the mass of the rope and the mass and friction of the pulley.

a. What is the magnitude of the normal force on the 5.0 kg mass? *(5 pts)*

$F_N = mg = (5.0 \text{ kg})(9.8\text{ms}^{-2}) =$

49 N

b. What is the magnitude of the force of friction on the 5.0 kg mass? *(5 pts)*

$F_K = \mu F_N = (0.20)(49\text{N})$

9.8 N

c. What is the magnitude of the acceleration of the 2.0 kg mass? *(9 pts)*

Forces: 5 kg: $T - F_K = m_5 a \Rightarrow T = F_K + m_5 a$, 2 kg: $m_2 g - T = m_2 a \Rightarrow m_2 g - F_K + m_5 a = m_2 a$

$$a = \frac{m_2 g - F_K}{m_5 + m_2} = \frac{19.6\text{N} - 9.8\text{N}}{5.0\text{kg} + 2.0\text{kg}} =$$

1.4 ms^{-2}

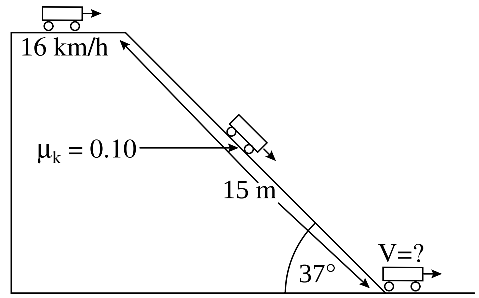
d. What is the tension in the rope *(6 pts)*?

$T = F_K + m_5 a = 9.8 \text{ N} + (5.0 \text{ kg})(1.4 \text{ ms}^{-1}) =$

17 N

PROBLEM 4

A 500.0 kg roller coaster starts with a speed of 16 km/hr. It then descends a hill, which is at an angle of 37° as shown and is 15 m long. The coefficient of kinetic friction between the roller coaster and the track is 0.10. *Note: the cart does not change speed at the instant it changes direction.*



a. What is the magnitude of the normal force on the roller coaster during its descent? *(5 pts.)*

$F_N = mg\cos37° = (500.0\text{ kg})(9.8\text{ms}^{-2})(0.80) = 3.91\text{x}10^2\text{ N}$

3,900 N

b. What is the magnitude of the force of friction during its descent? *(5 pts.)*

$F_K = \mu_K F_N = (0.10)(3.91\text{x}10^2\text{ N}) =$

390 N

c. What is the magnitude of the acceleration of the roller coaster during its descent? *(5 pts.)*

Define the x-axis along the direction of the hill:
$mg\sin37° - F_K = ma_x \Rightarrow mg\sin37° - \mu_K mg = ma_x \Rightarrow a_x = (\sin37° - (0.10)\cos37°) =$
$(9.8\text{ms}^{-2})(0.60 - (0.10)(0.80)) = 5.12\text{ ms}^{-1}$

5.1 ms^{-2}

d. What is the speed *in km/hr* of the roller coaster exactly at the end of the hill? *(5 pts.)*

$$v_{0x} = 16\frac{\text{km}}{\text{hr}}\left(\frac{1000\text{m}}{1\text{km}}\right)\left(\frac{1\text{hr}}{3600\text{s}}\right) = 16\frac{\text{km}}{\text{hr}}\left(\frac{1\text{m/s}}{3.6\text{km/hr}}\right) = 4.4\frac{\text{m}}{\text{s}}$$

$$v^2 = v_{0x}^2 + 2a_x(x - x_0) \Rightarrow v = \sqrt{(4.4\text{ms}^{-1})^2 + 2(5.1\text{ms}^{-2})(15\text{m})} = 13.1\frac{\text{m}}{\text{s}}\left(\frac{3.6\text{km/hr}}{1\text{m/s}}\right) =$$

47 km/hr

e. How far *in meters* does the roller coaster travel on the flat track after the hill? *(5 pts.)*

$$a = -\frac{F_K}{m} = -\frac{\mu mg}{m} = -\mu g = -0.98\frac{\text{m}}{\text{s}^2}, \quad v^2 = 0 = v_0^2 + 2a(x - x_0) \Rightarrow$$

$$x - x_0 = -\frac{v_0^2}{2a} = -\frac{(13.1\text{ms}^{-1})^2}{2(-0.98\text{ms}^{-2})} =$$

88 m