EXAM 2

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

TOTAL: __________

SOLUTION KEY

Possibly useful information:

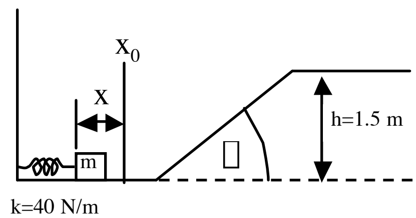
Acceleration due to gravity at the earth's surface: $g = 9.80 \text{ m/s}^2$

Moment of Inertia of a Solid Disc: $I = \frac{1}{2}MR^2$

First Name: ____________________ Last Name: _____________________ Section: __________ ■

PROBLEM 1

A block of mass m = 2.00 kg is pressed against a spring with force constant 40.0 N/m which is compressed a distance x from its equilibrium position x_0. The block is released from rest, and the spring causes the block to slide along a horizontal surface and up a ramp that is at angle θ = 30.0° to horizontal. The top of the ramp is a height h = 1.50 m above the horizontal surface where the block is initially resting. There is no friction anywhere between the block and the surface.



a. What is the potential energy of the spring if it is compressed by a distance x = 4.10 m? *(5 pts.)*

$$P.E. = \frac{1}{2}kx^2 = \frac{1}{2}(40.0N/m)(4.1m)^2 =$$

336 N-m

b. What is the compression distance x such that the block just makes it to the top of the ramp? *(5 pts.)*

Ball just makes it to top of ramp when $\frac{1}{2}kx^2 = mgh$

$$x = \sqrt{2mgh/k} = \sqrt{2(2.00kg)(9.8m/s^2)(1.50m)/(40.0N/m)} =$$

1.21 m

c. Under the conditions of part b, what is the magnitude of the velocity of the block when it is one third of the way up the ramp? *(5 pts.)*

Total mechanical energy is conserved $\Rightarrow \frac{1}{2}mv^2 + mgh/3 = mgh$

$$\Rightarrow v = \sqrt{\frac{2}{m}(2mgh/3)} = \sqrt{4gh/3} = \sqrt{4(9.8ms^{-2})(1.50m)/3} =$$

4.43 m/s

d. Under the conditions of part b, what is the power from the gravitational force on the block when it is one third of the way up the ramp? (5 pts)

Power = $\vec{F} \bullet \vec{v} = mgv\sin\theta = (2.00kg)(9.8m/s^2)(4.43m/s)(0.5) =$

43.4 W

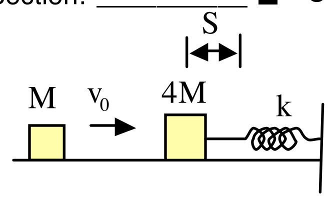
e. If instead the spring is initially compressed by 7.4m, what is the speed of the block when it reaches the top of the ramp? *(5 pts)*

Initial total mechanical energy is E = $kx^2/2$ = (40.0 N-m)$(7.4\text{ m})^2/2$ = 1095.2 J.
Total mechanical energy is conserved, so E = $mgh+mv^2/2$

$$\Rightarrow v = \sqrt{\frac{2}{m}(E - mgh)} = \sqrt{\frac{2}{(2.00kg)}(1095.2J - (2.00kg)(9.8ms^{-2})(1.5m))} =$$

32.7 m/s

PROBLEM 2



A block of mass M = 2.0 kg and velocity v_0 = 3.0 m/s moves to the right along a frictionless horizontal surface. It hits and sticks to a block of mass 4M=8.0 kg that is initially at rest. This second block is connected to a spring of force constant k=10.0 N/m that is in equilibrium before the collision.

a. Find the total mechanical energy before the collision. *(5 pts.)*

Initial PE=0, so E=(1/2)(2.0 kg)$(3.0\ \text{m/s})^2$ =

9.0 J

b. Find the velocity of the two blocks together just after the collision. *(5 pts.)*

Momentum conserved; $Mv_0=5Mv_f$, so v_f= (3.0 m/s)/5 =

0.60 m/s

c. What is the total mechanical energy of the two blocks together just after the collision? *(5 pts.)*

Note collision does not conserve energy. Spring not compressed yet, so potential energy is zero, and total mechanical energy is $(1/2)(5M)v_f^2 = (1/2)(10.0\ \text{kg})(0.60\ \text{m/s})^2$ =

1.8 J

d. What is the maximum distance S by which the spring is compressed? *(5 pts.)*

Maximum compression when blocks are stationary, and $kx^2/2$=1.8 J,

$$\Rightarrow x = \sqrt{2(1.8J)/(10.0N/m)} =$$

0.60 m

e. By what distance is the spring compressed at the instant when the velocity of the two blocks together is 0.50 m/s? *(5 pts.)*

After the collision the total mechanical energy is 1.8 J, so

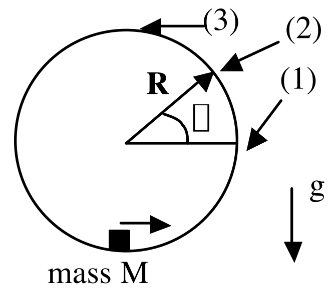
$$\tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 = E$$

$$\Rightarrow x = \sqrt{\tfrac{2}{k}\left(E - \tfrac{1}{2}mv^2\right)} = \sqrt{\frac{2}{(10.0N/m)}\left(1.8J - 0.5(10.0kg)(0.50m/s)^2\right)} =$$

0.34 m

PROBLEM 3

A mass M = 0.30 kg is sliding without friction on a vertical circular track of radius R = 2.0 m. The speed of the mass is not given, but at the bottom of the track, the normal force between the mass and the track is given to be 19.0 N upward. The point (1) is at a height R, the point (2) is where θ is 37°, and the point (3) is at the top of the track, at height 2R.



a. What is the magnitude of the velocity of the mass M at the bottom of the track? *(5 pts)*

$Mv^2/R = N - Mg$, so

$$v=\sqrt{\frac{R}{M}(N-Mg)}=\sqrt{\frac{2.0m}{0.3kg}(19.0N-(0.3kg)(9.8m/s^2))}=$$

10 m/s

b. What is the total mechanical energy of the mass M at point (1)? *(5 pts)*

Mechanical energy is conserved, so can evaluate it where the potential energy is zero, at which point the mechanical energy $E = Mv^2/2 = (0.30\ kg)(10.3\ m/s)^2/2 =$

16 J

c. What is the magnitude of the velocity of the mass M at point (1)? *(5 pts)*

$Mv^2/2 + MgR = E$

$$\Rightarrow\ v=\sqrt{\frac{2}{M}(E-MgR)}=\sqrt{\frac{2}{0.30kg}\left(16.1J-(0.30kg)(9.8ms^{-2})(2.0m)\right)}=$$

8.2 m/s

d. What is the magnitude of the velocity of mass M at point (2) *(5 pts)*?

Height is now $h=R(1+\sin\theta)=(2.0\ m)(1+\sin 37°) = 3.20\ m$

$E = Mv^2/2 + Mgh$

$$\Rightarrow\ v=\sqrt{\frac{2}{M}(E-Mgh)}=\sqrt{\frac{2}{0.3kg}\left(16.1J-(0.3kg)(9.8ms^{-2})(3.2m)\right)}=$$

6.7 m/s

e. What is the magnitude of the normal force on mass M at point (3)? *(5 pts)*

First find acceleration of the mass. At point (3), height is 2R, so

$$\Rightarrow\ v=\sqrt{\frac{2}{M}(E-Mg(2R))}=\sqrt{\frac{2}{0.3kg}\left(16.1J-(0.3kg)(9.8ms^{-2})(4.0m)\right)}=5.35m/s$$

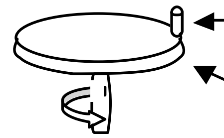
$N + Mg = Mv^2/R$

$$\Rightarrow\ N=M\left(\frac{v^2}{R}-g\right)=(0.30kg)\left(\frac{(5.35m/s)^2}{2.0m}-9.8m/s^2\right)=$$

1.4 N

PROBLEM 4

A 31 g point mass rests at the edge of a spinning disk ($I_{disk}=MR^2/2$) of mass M=1.4 kg and radius R=0.60 m. The static coefficient of friction between the mass and the disk is 0.75, and the kinetic coefficient of friction between the mass and the disk is 0.62.



a. What is the maximum rate in revolutions per second that the disk can rotate, without the mass sliding off the disk? *(5 pts.)*

The horizontal friction force that causes mass to accelerate inward has maximum magnitude $\mu_s Mg$, so maximum acceleration of the mass is $\mu_s Mg/M = \mu_s g$. Since acceleration $a=\omega^2 R$, the maximum angular velocity ω_m is $(a/R)^{1/2}$. The maximum number of revolutions per second $f_m = \omega_m/(2\pi)$, so

$$f_m = \frac{\sqrt{\mu_s g/R}}{2\pi} = \frac{1}{2\pi}\sqrt{(0.75)(9.8m/s^2)/(0.60m)} =$$

0.56 revolutions/s

b. What is the kinetic energy of the point mass at this maximum rotation rate? *(5 pts.)*

Acceleration $v^2/R = \mu_s g$, so $v^2 = \mu_s gR$. $K= mv^2/2$, so

$K= m\mu_s gR /2=(0.031 \text{ kg})(0.75)(9.8 \text{ m/s}^2)(0.60 \text{ m})/2 =$

0.068 J

c. What is the kinetic energy of the disk alone at this maximum rotation rate? *(5 pts.)*

Kinetic energy $K=I\omega^2$ and $I=MR^2/2$, so $K= MR^2\omega^2/4$. The angular velocity ω = 3.5 radians/s, so

$K=(1.4 \text{ kg})(0.60 \text{ m})^2(3.5 \text{ s}^{-1})^2/4=$

1.5 J

d. A second disk, also with mass 1.4 kg and radius 0.60 m (and no mass on top) is initially rotating at 10.0 revolutions/sec. A constant force is applied at the edge of the disk directed opposite to the motion of the edge, causing the disk to stop rotating after 10.0 seconds. What is the magnitude of the torque caused by the force? *(5 pts.)*

Torque $\tau= I\alpha$, where $\alpha=d\omega/dt$. Initial angular velocity $\omega=2\pi f=62.8 \text{ s}^{-1}$, acceleration is constant in time, so $\alpha=(62.8 \text{ s}^{-1})/(10.0 \text{ s})=6.28 \text{ s}^{-2}$. Since $I=MR^2/2=(1.4 \text{ kg})(0.60 \text{ m})^2/2 = 0.252 \text{ kg-m}^2$,

$\tau= (0.252 \text{ kg-m}^2)(6.28 \text{ s}^{-2}) =$

1.6 N-m

e. What is the power exerted by the force in part d at the instant when the disk is rotating at 3.5 revolutions per second? *(5 pts.)*

Power $P = \tau\omega = (1.6 \text{ N-m})(2\,\pi\,3.5 \text{ s}^{-1}) =$

35 W