

## FINAL EXAM

**Print your name and section clearly on all nine pages.** (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided.** Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points.** In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 8.5" x 11" sheet and no other references. The exam lasts exactly two hours.

*(Do not write below)*

### SCORE:

Problem 1: \_\_\_\_\_

Problem 2: \_\_\_\_\_

Problem 3: \_\_\_\_\_

Problem 4: \_\_\_\_\_

Problem 5: \_\_\_\_\_

Problem 6: \_\_\_\_\_

Problem 7: \_\_\_\_\_

Problem 8: \_\_\_\_\_

**TOTAL:** \_\_\_\_\_

Possibly useful information:

Acceleration due to gravity at the earth's surface:  $g = 9.80 \text{ m/s}^2$

Gravitational Constant:  $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

1 calorie = 4.186 Joules

1 atm =  $1.013 \times 10^5 \text{ Pa}$

Universal Gas Constant:  $R = 8.314 \text{ J}/(\text{mol}\cdot\text{K})$

Stefan-Boltzmann Constant:  $\sigma = 5.669 \times 10^{-8} \text{ W}/\text{m}^2\text{K}^4$

Avogadro's Number:  $N_A = 6.022 \times 10^{23} \text{ molecules}/\text{mole}$

Boltzmann's Constant:  $k_b = 1.38 \times 10^{-23} \text{ J}/\text{K}$

**PROBLEM 1**

4.00 moles of  $N_2$  (an ideal *diatomic* gas with  $\gamma = 1.40$  and molecular weight 28.0 g/mole) undergo a temperature increase from 300.0 K to 360.0 K at constant pressure.

a. How much heat in Joules was added to the gas? (5 pts.)

b. By how much did the internal energy of the gas increase? (5 pts.)

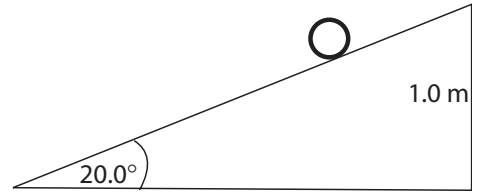
c. How much work was done by the gas? (5 pts.)

d. By how much did the translational kinetic energy of the gas increase? (5 pts.)

e. What is the rms velocity of the  $N_2$  molecules at 360 K? (5 pts.)

**PROBLEM 2**

A 3.0 kg uniform thin circular ring ( $I=MR^2$ ) is released from rest at the top of a 1.0 m high ramp inclined at an angle of  $20.0^\circ$  and rolls down the ramp without slipping.



a. Find the total kinetic energy of the ring when it reaches the bottom of the ramp. (5 pts.)

b. What percentage of this energy is in translation of the center of mass? (5 pts.)

c. Find the speed of the center of mass when the ring reaches the bottom part of the ramp. (5 pts.)

d. What is the magnitude of the acceleration of the center of mass when the ring reaches the bottom part of the ramp? (5 pts.)

e. How much time does it take for the ring to reach the bottom of the ramp? (5 pts.)

**PROBLEM 3**

A fire hose is attached to a hydrant outlet at ground level. The outlet has an area of  $0.12 \text{ m}^2$ , and provides  $4.0 \times 10^5 \text{ Pa}$  of water pressure with a water speed of  $5.0 \text{ m/s}$ . The mass density of water is  $1000.0 \text{ kg/m}^3$ . Assuming an idea flow:

a. Find the volume flow rate in  $\text{m}^3/\text{s}$  of the water exiting the hydrant. (5 pts.)

b. The hydrant outlet is attached to a hose with a nozzle with an area of  $0.088 \text{ m}^2$  at the same height as the hydrant outlet. What is the speed of the water as it exits the nozzle? (5 pts.)

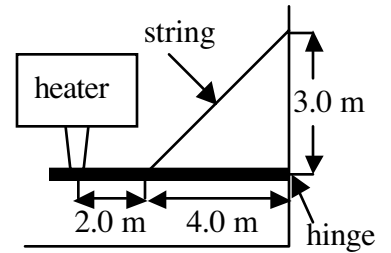
c. In part b, what is the gauge pressure just inside the end of the nozzle? (5 pts.)

d. As the firefighter holding the hose from part b climbs a ladder, the speed of the water coming out of the hose decreases. What is the maximum height that the nozzle can be at which water still flows? (5 pts.)

e. Another firefighter holds a different hose with water exiting the nozzle horizontally at a height of  $12.5 \text{ m}$  above the ground and speed of  $5.5 \text{ m/s}$ . Ignoring air resistance, how far from the point on the ground directly below the nozzle does the water hit the ground? (5 pts.)

**PROBLEM 4**

a. A pine log cabin has a wall area of  $60.0 \text{ m}^2$  and walls  $0.14 \text{ m}$  thick. The ceiling and floor are well-insulated, so you may assume that all heat is lost through the walls. The thermal conductivity of pine is  $k_p = 0.21 \text{ Wm}^{-1}\text{K}^{-1}$ . A heater of mass  $5.5 \text{ kg}$  is resting on a rigid massless shelf supported by a hinge and a massless string, as shown. When set on medium, the heater supplies  $1800.0 \text{ W}$  continuously to the cabin interior.



a. What is the tension in the string? (5 pts.)

b. What is the magnitude of the force on the hinge?

c. If the outside temperature is  $0.0^\circ \text{ C}$ , what is the temperature inside the cabin? (5 pts.)

d. The interior walls of the cabin are sprayed with a layer of insulating foam  $0.02 \text{ m}$  thick, with thermal conductivity  $\kappa_f = 0.015 \text{ Wm}^{-1}\text{K}^{-1}$ . If the outside temperature is  $0.0^\circ \text{ C}$  and the heater is set on low ( $1000.0 \text{ W}$  continuously), what is the temperature at the foam/wood interface? (5 pts.)

e. In part d, what is the temperature inside the cabin? (5 pts.)

**PROBLEM 5**

A person adds 0.0900 kg of ice at  $0.00^\circ\text{C}$  to an insulated mug containing tea at  $50.0^\circ\text{C}$ . The heat of fusion for ice is  $L_{\text{ICE}} = 333\text{ kJ/kg}$ , and the specific heat of water (and tea) is  $c_w = 4.20\text{ kJkg}^{-1}\text{K}^{-1}$ . The mass density of ice  $\rho_i$  is  $0.931\text{ gm/cm}^3$ , while the mass density of water  $\rho_w$  is  $1.00\text{ gm/cm}^3$ .

a. What fraction of the ice is above the surface of the water?

b. Find the smallest amount of grams of tea at 50.0 degrees C that will just melt all the ice, leaving only liquid at 0.00 degrees C. (5 pts.)

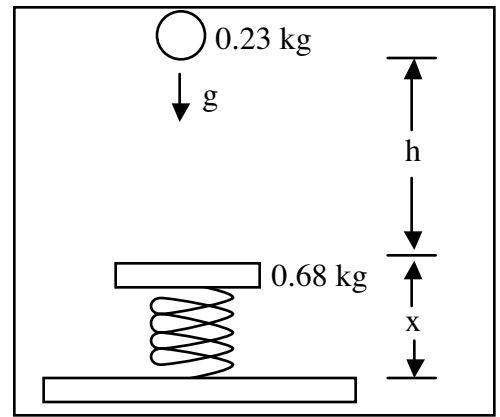
c. If the mug has 0.400 kg of tea before the 0.0900 kg of ice is added, what is the final temperature  $T_f$  in degrees C, of the mixture when it reaches equilibrium? (5 pts.)

d. A straw with thermal expansion coefficient of  $3.200 \times 10^{-4}\text{ K}^{-1}$  is a hollow cylinder of inner diameter 0.530 cm when sitting in tea at 50.00 degrees C. What is the inner diameter of the straw when the liquid is cooled down to 5.00 degrees C? (5 pts.)

e. A person moves the 0.490 kg of cool tea and the 0.100 kg mug containing it from a tabletop 1.00 m above the floor to a shelf 1.90 m above the floor. How much work was done by the gravitational force on the mug of tea during this move? (5 pts.)

**PROBLEM 6**

A ball of mass  $m=0.23$  kg starts from rest and after falling a distance  $h$  accelerated by gravity, it is moving at a speed of  $8.5$  m/s when it hits and sticks to a platform of mass  $M=0.68$  kg that just before the collision is at rest, supported by a massless, frictionless spring with spring constant  $k=55.5$ N/m.



a. What is the distance  $h$ ? (5 pts.)

b. What is the velocity of the ball and platform at the instant just after the ball collides and sticks to the platform? (5 pts.)

c. Now suppose that at the instant of impact of the ball with the platform, the platform is a distance  $x=0.44$  m above the floor. The spring then compresses, decreasing  $x$  until it hits a minimum value, and the platform then undergoes oscillatory motion. What is the period of the oscillation? (5 pts.)

d. What is the potential energy of the spring at the instant of impact in part c? (5 pts.)

e. After a very long time the ball-platform-spring system comes to rest. What is the value of  $x$  at which this occurs? (5 pts.)

**PROBLEM 7**

A planet has mass  $M=7.0 \times 10^{21}$  kg, uniform mass density, and radius  $R=2.0 \times 10^6$  m.

a. What is the acceleration of gravity on the planet's surface? (5 pts.)

b. What is the average mass density of the planet? (5 pts.)

c. A mass of 8.0 kg is in a circular orbit around the planet at an altitude  $R= 2.0 \times 10^6$  m above the surface. What is its kinetic energy? (5 pts.)

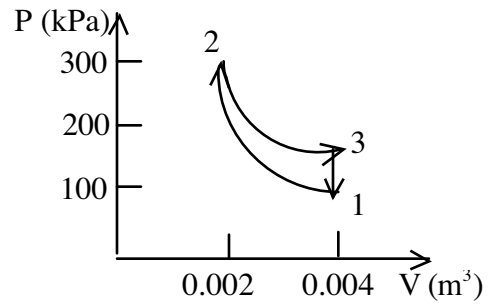
d. A small object hangs motionless with respect to the planet at a very large (infinite) distance from the planet. A very slight (negligible) nudge towards the planet causes it to be captured by the planet's gravity. The object eventually crashes onto the planet's surface. What is its speed when it lands? (5 pts.)

e. A person on the planet's surface fashions a simple pendulum from a (nearly) massless 2.0 m length of string and a lead weight. What is the pendulum period for small oscillations about the vertical? (5 pts.)



**PROBLEM 8**

An engine operates in the following cycle, using a monatomic ideal gas as its working substance: (1→2): The gas starts in initial state  $(P_1, V_1, T_1) = (100.0 \text{ kPa}, 0.004 \text{ m}^3, 300.0 \text{ K})$  and is compressed adiabatically to  $(P_2, V_2, T_2) = (317 \text{ kPa}, 0.002 \text{ m}^3, 475.5 \text{ K})$  by a piston. (2→3): The gas expands isothermally to  $(P_3, V_3, T_3) = (158.5 \text{ kPa}, 0.004 \text{ m}^3, 475.5 \text{ K})$ , doing work on the piston. (3→1): The gas then cools at constant volume back to the state  $(P_1, V_1, T_1)$ . The P-V diagram for the process is shown in the figure.



a. What is the change in entropy of the gas along the (2→3) leg? (5 pts.)

b. What is  $Q_{2,3}$ , the amount of heat absorbed by the engine in the (2→3) leg? (5 pts.)

c. What is  $Q_{3,1}$ , the amount of heat ejected by the engine in the (3→1) leg? (5 pts.)

d. Using the First Law of Thermodynamics and the heats from parts b and c, calculate the work done by the gas per (1→2→3→1) cycle. (5 pts.)

e. Find the efficiency  $\epsilon$  of this engine. (5 pts.)