EXAM 2

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

TOTAL: __________

SOLUTION KEY

Possibly useful information:

Acceleration due to gravity at the earth's surface: $g = 9.80\ \text{m/s}^2$

$$\text{For } ax^2 + bx + c = 0,\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PROBLEM 1

A rider on a bicycle (total mass 85.0 kg) travels on a horizontal flat circular track of radius 25.0 m at a constant speed of 9.00 m/s.

a. What is the force of friction exerted by the road on the bicycle? *(5 pts.)*

$$f = F_C = \frac{mV^2}{R} = \frac{(85.0kg)(9.00\,m/s)^2}{25.0m} =$$

275 N

b. What is the magnitude of the total force exerted by the road on the bicycle? *(5 pts.)*

$$N = mg = (85.0\ kg)(9.8\ m/s) = 833\ N, \quad N \perp f \Rightarrow F = \sqrt{f^2 + N^2} = \sqrt{(275N)^2 + (833N)^2} =$$

877 N

c. The bicyclist now brakes with a uniform force and stops after traveling 123 m around the track. What is the magnitude of the braking force? *(5 pts)*

$$Fd = W = -\Delta K = K_i - K_f = \tfrac{1}{2}mv^2 - 0 \Rightarrow F = \frac{mv^2}{2d} = \frac{(85.0kg)(9.00\,m/s)}{2(123m)} =$$

28.0 N

d. What is the braking power when the brakes are first applied? *(5 pts.)*

$$P = F \bullet V = (28.0\ N)(9.00\ m/s) =$$

252 W

e. Assuming the bicycle wheels of radius 35.6 cm do not skid on the road, what is their angular acceleration while braking? *(5 pts.)*

$$F = ma \Rightarrow a = F/m = (28.0\ N)/(85.0\ kg) = .329\ m/s^2,\ \alpha = a/r = (9.329\ m/s^2)/(.356\ m) =$$

$0.925\ rad/s^2$

PROBLEM 2

Bill, who has mass 83 kg, and Jane, who has mass 55 kg are skating on a frictionless surface. Bill is gliding East at 6.2 km/hr and Jane is gliding North at 7.8 km/hr. They collide and stick together, and then continue to glide on the frictionless surface.

a. What is the direction (degrees with respect to which axis) of their velocity after the collision? *(5 pts.)*

$$m_B v_B = MV\cos\theta,\ m_J v_J = MV\sin\theta \Rightarrow \tan\theta = \frac{m_J v_J}{m_B v_B} = \frac{(55\text{kg})(7.8\,\text{km/hr})}{(83\text{kg})(6.2\,\text{km/hr})} = 0.834 \Rightarrow \theta = 39.8^\circ$$

40.° north of east

b. What is the magnitude of their velocity (in km/hr) after the collision? *(5 pts.)*

$$V = \frac{m_J v_J}{M\sin\theta} = \frac{(55\text{kg})(7.8\,\text{km/hr})}{(83\text{kg}+55\text{kg})(\sin 39.8^\circ)} =$$

4.9 km/hr

c. How much kinetic energy was lost in the collision? *(5 pts)*

$$4.86\frac{\text{km}}{\text{hr}}\times\frac{1000\,\text{m/km}}{3600\,\text{s/hr}} = 1.35\frac{\text{m}}{\text{s}},\ 6.2\frac{\text{km}}{\text{hr}}\times\frac{1000\,\text{m/km}}{3600\,\text{s/hr}} = 1.72\frac{\text{m}}{\text{s}},\ 7.8\frac{\text{km}}{\text{hr}}\times\frac{1000\,\text{m/km}}{3600\,\text{s/hr}} = 2.17\frac{\text{m}}{\text{s}}$$

$$K_b = \tfrac{1}{2}m_B v_B^2 + \tfrac{1}{2}m_J v_J^2 = \tfrac{1}{2}(83\text{kg})(1.72\,\text{m/s})^2 + \tfrac{1}{2}(55\text{kg})(2.17\,\text{m/s})^2 = 123\text{J} + 129\text{J} = 252\text{J}$$

$$K_a = \tfrac{1}{2}MV^2 = \tfrac{1}{2}(83\text{kg}+55\text{kg})(1.35\,\text{m/s})^2 = 126\text{J},\quad K_a - K_b = 252\text{J} - 126\text{J} = 126\text{J}$$

130 J

d. If they exerted a constant force on each other for 1.2 seconds during the collision, what was the magnitude of this force? *(5 pts.)*

$$\Delta P_X = m_B(V\cos\theta - V_B) = (83\text{kg})(1.35\,\text{m/s}\cdot\cos 39.8^\circ - 1.72\,\text{m/s}) = -56.7\,\text{kgm/s}$$

$$\Delta P_Y = m_B V\sin\theta = (83\text{kg})(1.35\,\text{m/s}\cdot\sin 39.8^\circ) = 71.7\,\text{kgm/s}$$

$$\Delta P = \sqrt{\Delta P_X^2 + \Delta P_Y^2} = \sqrt{(-56.7\,\text{kgm/s})^2 + (71.7\,\text{kgm/s})^2} = 91.4\,\text{kgm/s},\quad F = \Delta P/\Delta t = (91.4\,\text{kgm/s})/(1.2\text{s}) =$$

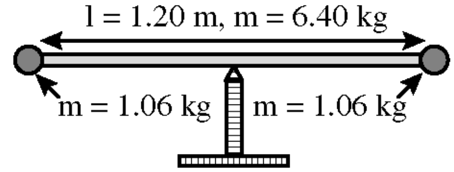
76 N

e. What was the average power dissipated during the 1.2 seconds? *(5 pts.)*

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta K}{\Delta t} = \frac{126\text{J}}{1.2\text{s}} = 105\,\text{W}$$

110 W

PROBLEM 3



A thin rod ($I = ML^2/12$) of length 1.20 m and mass 6.40 kg has two balls of mass 1.06 kg, one attached to each end as shown. The rod rotates in a horizontal plane about a vertical axis through a pivot at its midpoint. At one moment it rotates at 39.0 revolutions/sec. Because of a constant frictional torque, it comes to rest 32.0 sec later.

a. What is the angular acceleration of the rod (in rad/sec^2)? *(5 pts)*

$$0 = \omega_0 + \alpha t \Rightarrow \alpha = \frac{-\omega_0}{t} = \frac{-(39.0\,\text{rev/s})(2\pi\,\text{rad/rev})}{32.0\text{s}} =$$

-7.66 rad/sec^2

b. What is the total moment of inertia about the pivot of the balls and rod? *(5 pts)*

$$I = \frac{ML^2}{12} + 2m\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{mL^2}{2} = \frac{(6.40\text{kg})(1.20\text{m})^2}{12} + \frac{(1.06\text{kg})(1.20\text{m})^2}{2} =$$

1.53 kgm^2

c. What is the magnitude of the retarding torque exerted by friction? *(5 pts.)*

$$\tau = I\alpha = (1.53\text{kgm}^2)(-7.66\,\text{rad/s}^2) =$$

11.7 Nm

d. What is the total work done by friction in the 32.0 sec? *(5 pts)*

$$W = \Delta K = K_f - K_i = 0 - K_i = -\tfrac{1}{2}I\omega_0^2 = -\tfrac{1}{2}(1.53\text{kgm}^2)\left[(39.0\,\text{rev/s})(2\pi\,\text{rad/rev})\right]^2 =$$

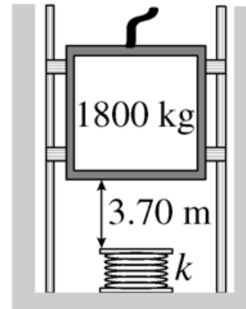
-4.59×10^{4}J

e. How many revolutions were made in the 32.0 sec? *(5 pts.)*

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = 2\pi(39.0\,\text{rad/s})(32.0\text{s}) + \tfrac{1}{2}(-7.66\,\text{rad/s}^2)(32.0\text{s})^2 = 3920\text{rad} \times (1\text{rev}/2\pi\text{rad}) =$$

624 rev

PROBLEM 4



The cable of a 1800.0 kg elevator snaps when it is at rest 3.70 m above a spring with spring constant k = 150.0 kN/m. The elevator also instantly clamps to safety rails providing a constant force of kinetic friction of 4.40 kN opposing the motion of the elevator. *(Ignore effects of static friction on the elevator at rest).*

a. What is the speed of the elevator just before it hits the spring? *(6 pts.)*

$$W = mgd - fd = \Delta K = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2d(mg-f)}{m}} = \sqrt{\frac{2(3.70\text{m})\left[(1800\text{kg})(9.8\,\text{m/s}^2) - 4{,}400\text{N}\right]}{1800\text{kg}}} =$$

7.38 m/s

b. What is the maximum compression of the spring? *(7 pts.)*

$$-fx = \tfrac{1}{2}kx^2 - \left(mgx + \tfrac{1}{2}mv^2\right) \Rightarrow -4400\tfrac{\text{N}}{\text{m}}x = \tfrac{1}{2}(150{,}000\text{N})x^2 - (1800\text{kg})\left(9.8\tfrac{\text{m}}{\text{s}^2}\right)x - \tfrac{1}{2}(1800\text{kg})\left(7.38\tfrac{\text{m}}{\text{s}}\right)^2$$

$$\Rightarrow 75{,}000x^2 - 13{,}240x - 49{,}018 = 0 \Rightarrow x = \frac{-13{,}240 \pm \sqrt{(13{,}240)^2 + 4(75{,}000)(49{,}018)}}{2(75{,}000)} =$$

$-0.725\text{m} \Leftarrow$ wrong direction or 0.902m

0.902 m

c. What maximum distance from the top of the spring at the point of maximum compression will the elevator bounce up the shaft? *(6 pts.)*

$$\tfrac{1}{2}kx^2 - mgy - fy = \Delta E = 0 \Rightarrow y = \frac{kx^2}{2(mg+f)} = \frac{(150{,}000\,\text{N/m})(0.902\text{m})^2}{2\left[(1800\text{kg})(9.8\,\text{m/s}^2) + 4400\text{N}\right]} =$$

2.77 m

d. What total distance does the elevator move after the cable snaps before it comes to rest? *(6 pts.)*

$$\Delta E = \tfrac{1}{2}kx^2 - mg(d+x) = W = -f\ell \text{ (elevator at rest } \Rightarrow kx = mg)$$

$$\ell = \frac{mg}{f}\left(d + \frac{mg}{2k}\right) = \frac{(1800\text{kg})(9.8\,\text{m/s}^2)}{4400\text{N}}\left(3.7\text{m} + \frac{(1800\text{N})(9.8\,\text{m/s}^2)}{2(150{,}000\,\text{N/m})}\right) =$$

15.1 m