November 22, 2004 Physics 201

EXAM 3

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

TOTAL: __________

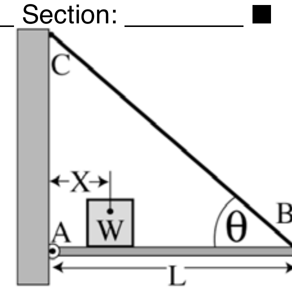
SOLUTION KEY

Possibly useful information:

Acceleration due to gravity at the earth's surface: $g = 9.80\ \text{m/s}^2$
Atmospheric pressure at sea level = 1.01×10^5 Pa = $1.01 \times 10^5\ \text{N/m}^2$
Gravitational constant: $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$
$\rho(\text{water}) = 1.00 \times 10^3\ \text{kg/m}^3 = 1.00\ \text{g/cm}^3$
1 Liter = $10^{-3}\ \text{m}^3$

First Name: ________________________ Last Name: _________________________ Section: ________ ■

PROBLEM 1



A uniform bar of length L = 3.0 m and weight 200.0 N is pinned to a vertical wall at point A and held in place horizontally by a wire from C to B at an angle of θ = 30.0°, with respect to the bar. The 3.6 mm diameter wire can withstand a maximum tension of 500.0 N before it breaks. A weight W = 300 N is moved a distance X away from the wall along the bar.

a. What is the maximum distance X before the wire breaks? *(7 pts.)*

$$\sum \tau = 0 \rightarrow T_{max} L \sin\theta = WX_{max} + W_b\left(L/2\right) \rightarrow X_{max} = \frac{\left(T_{max}\sin\theta - W_b/2\right)L}{W}$$

$$= \frac{\left((500\text{N})(\sin 30°) - 200\text{N}/2\right)3.0\text{m}}{300\text{N}} =$$

1.5 m

b. With the weight at this position X, what is the horizontal component of the force exerted on the bar by the pin at A? *(6 pts.)*

$\Sigma F_h = F_{Ah} - T_{max}\cos\theta = 0 \rightarrow F_{Ah} = T_{max}\cos\theta = (500\text{ N})\cos 30° = 433\text{ N}$

430 N

c. With the weight at this position X, what is the vertical component of the force exerted on the bar by the pin at A? *(6 pts.)*

$\Sigma F_v = F_{Av} + T_{max}\sin\theta - W - W_b = 0 \rightarrow F_{Av} = W + W_b - T_{max}\sin\theta = 300\text{ N} + 200\text{ N} - 500\text{ N}\sin 30° =$

250 N

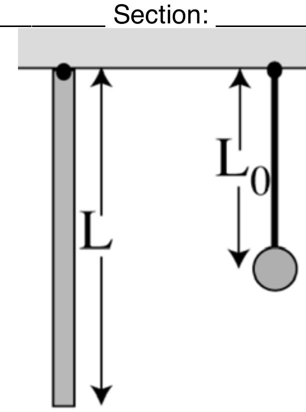
d. With the weight at this position X, if the Young's Modulus of the wire is 2.0×10^{11} N/m^2, assuming it stretches elastically before it breaks, how much does the wire length increase over its unstressed length? *(6 pts.)*

$$L = \frac{3.0\text{m}}{\cos 30°} = 3.46\text{m}, \quad \frac{F}{A} = Y\frac{\Delta L}{L} \rightarrow \Delta L = \frac{FL}{AY} = \frac{(500\text{N})(3.46\text{m})}{\pi\left(1.8\times10^{-3}\text{m}\right)^2\left(2.0\times10^{11}\text{ N/m}^2\right)} = 8.5\times10^{-4}\text{ m}$$

0.85 mm

PROBLEM 2

A uniform rod (I of the rod about its end = $ML^2/3$) of length L = 1.6 m and mass M = 2.3 kg is suspended as a physical pendulum from one end as shown. Assume that all oscillations of the rod are small.



a. What is the period of its oscillation? *(5 pts.)*

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mg\left(\frac{1}{2}L\right)}} = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2(1.6\,\text{m})}{3(9.8\,\text{m/s}^2)}} = 2.07\text{s}$$

2.1 s

b. What length, L_0, would a simple pendulum of mass M = 2.3 kg (pictured on the right) need to have the same period? *(5 pts.)*

$$2\pi\sqrt{\frac{L_0}{g}} = 2\pi\sqrt{\frac{2L}{3g}} \rightarrow L_0 = \frac{2}{3}L = \frac{2}{3}(1.6\text{m}) =$$

1.1 m

c. The rod is displaced 0.050 radians from vertical and released. What is its maximum angular speed? *(5 pts.)*

$$\theta = \theta_{max}\cos\omega t \rightarrow \Omega = -\omega\theta_{max}\sin\omega t \rightarrow \Omega_{max} = \omega\theta_{max} = \frac{2\pi}{T}\theta_{max} = \frac{2\pi(0.050\text{rad})}{2.07\text{s}} =$$

0.15 rad/s

d. For the same conditions as part c, what is the maximum angular acceleration of the rod? *(5 pts.)*

$$\alpha = -\omega^2\theta_{max}\cos\omega t \rightarrow \alpha_{max} = \omega^2\theta_{max} = \left(\frac{2\pi}{T}\right)^2\theta_{max} = \frac{4\pi^2(0.050\text{rad})}{(2.07\text{s})^2} =$$

0.46 rad/s^2

e. The suspended rod is now placed in an elevator accelerating up at 2.0 m/s^2. What is the period of the rod's oscillation? *(5 pts.)*

$$T = 2\pi\sqrt{\frac{2L}{3(g+a)}} = 2\pi\sqrt{\frac{2(1.6\text{m})}{3(9.8\,\text{m/s}^2 + 2.0\,\text{m/s}^2)}} =$$

1.9 s

PROBLEM 3

A 220.0 kg satellite starts in circular orbit 640.0 km above the surface of the earth (radius = 6.37×10^6 m, mass = 5.98×10^{24} kg). After its first orbit, it begins to lose energy due to a constant frictional force at a rate of 1.40×10^5 J/revolution. Assume that the orbit remains circular at all times.

a. What is the speed of the satellite during the first orbit? *(5 pts.)*

r = 6.37×10^6 m + 640×10^3 m = 7.01×10^6 m

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{7.01\times10^{6}\,\mathrm{m}}} =$$

7.54×10^3m/s

b. What is the period (in minutes) of the first orbit? *(5 pts.)*

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(7.01\times10^{6}\,\mathrm{m}\right)}{7.54\times10^{3}\,\mathrm{m/s}} = 5.84\times10^{3}\,\mathrm{s} =$$

97.4 min

c. Exactly 1500 orbits after its first orbit, find the distance of the satellite above the earth's surface. *(5 pts.)*

$$E = \frac{-GMm}{2r} \rightarrow r' = \frac{-GMm}{2E'} = -GMm\Big/2\left(\frac{-GMm}{2r} - \Delta E\right) = \left(\frac{1}{r} + \frac{2\Delta E}{GMm}\right)^{-1} =$$

$$\left(\frac{1}{7.01\times10^{6}\,\mathrm{m}} + \frac{2(1500)\left(1.4\times10^{5}\,\mathrm{J}\right)}{\left(6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)(220\mathrm{kg})}\right)^{-1} = 6.78\times10^{6}\,\mathrm{m}$$

$$h = 6.78\times10^{6}\,\mathrm{m} - 6.38\times10^{6}\,\mathrm{m} = 4.0\times10^{5}\,\mathrm{m}$$

400 km

d. 1500 orbits after its first orbit, what is the period (in minutes) of the satellite's orbit? *(5 pts.)*

$$T' = 2\pi\sqrt{\frac{r'^3}{GM}} = \sqrt{\frac{\left(6.78\times10^{6}\,\mathrm{m}\right)^3}{\left(6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}} = 5{,}554\mathrm{s} =$$

92.6 min

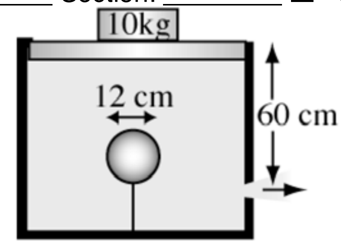
e. How much did the angular momentum of the satellite change during the 1500 orbits? *(5 pts.)*

$$v' = \frac{2\pi r'}{T'} = \frac{2\pi\left(6.78\times10^{6}\,\mathrm{m}\right)}{5{,}554\mathrm{s}} = 7.67\times10^{3}\,\mathrm{m/s}, \quad \Delta L = L' - L = mv'r' - mvr = m\left(v'r' - vr\right)$$

$$= (220\mathrm{kg})\left(\left(7.67\times10^{3}\,\mathrm{m/s}\right)\left(6.78\times10^{6}\,\mathrm{m}\right) - \left(7.54\times10^{3}\,\mathrm{m/s}\right)\left(7.01\times10^{6}\,\mathrm{m}\right)\right) =$$

-1.88×10^{11} kgm/s

PROBLEM 4



A tank of cross-sectional area of 0.070 m^2 is filled with water. A tightly fitting freely sliding piston with total mass 10.0 kg rests on top of the water. Inside the tank, an evacuated spherical shell of mass 0.30 kg and diameter 12.0 cm is attached with a string to the bottom of the tank. A circular hole of diameter 1.5 cm is opened at a depth of 60.0 cm below the piston. Note the area of the hole is much smaller than the piston area.

a. What is the tension in the string? *(5 pts.)*

$$T = \tfrac{4}{3}\pi R^3 \rho_w g - m_{sp} g = \tfrac{4}{3}\pi (0.06\,\mathrm{m})^3 \left(10^3\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right) - (0.30\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right) =$$

5.9 N

b. What is the gauge pressure at the surface of the water? (5 pts.)

$$p_1 A = mg \rightarrow p_1 = \frac{mg}{A} = \frac{(10\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)}{0.07\,\mathrm{m^2}} = 1400\,\mathrm{N/m^2} =$$

1400 Pa

c. What is the gauge pressure at the depth of 60 cm below the water level of the tank? *(5 pts.)*

$$p' = \rho_w g h + p_1 = \left(10^3\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)(0.60\,\mathrm{m}) + 1400\,\mathrm{N/m^2} = 5{,}880\,\mathrm{N/m^2} + 1400\,\mathrm{N/m^2} = 7{,}280\,\mathrm{N/m^2} =$$

7,300 Pa

d. What is the velocity of the fluid emerging from the hole? (5 pts.)

$$p_0 + p_1 + \rho g h = p_0 + \tfrac{1}{2}\rho v^2 \rightarrow v^2 = 2(p_1 + \rho g h)/\rho \rightarrow$$

$$v = \sqrt{2\left((p_1/\rho) + gh\right)} = \sqrt{2\left(\left(1400\,\mathrm{N/m^2}/10^3\,\mathrm{N/m^2}\right) + \left(9.8\,\mathrm{m/s^2}\right)(0.6\,\mathrm{m})\right)} =$$

3.8 m/s

e. How many liters per second emerge from the hole? *(5 pts.)*

Flux = VA_{hole}=(3.8m/s)(π(0.0075m)^2) = 6.7×10^{-4} m^3/s =

0.67 L/s