# FINAL EXAM

**Print your name and section <u>clearly</u> on all <u>nine</u> pages.** (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 8.5" x 11" sheet and no other references. The exam lasts exactly two hours.

*(Do not write below)*

**SCORE:**

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

Problem 5: __________

Problem 6: __________

Problem 7: __________

Problem 8: __________

**SOLUTION KEY**

**TOTAL:** __________

Possibly useful information:

Acceleration due to gravity at the earth's surface: $g = 9.80$ m/s$^2$
Gravitational Constant: G =6.67 x $10^{-11}$ Nm$^2$/kg$^2$
1 calorie = 4.186 Joules, 1 atm = 1.013 × $10^5$ Pa, 0 °C = 273.1 °K
Universal Gas Constant: R = 8.314 J/(mol·K)
Stefan-Boltzmann Constant: $\sigma$ = 5.669 x $10^{-8}$ W/m$^2$K$^4$
Avogadro's Number: $N_A$=6.022 x $10^{23}$ molecules/mole
Boltzmann's Constant: $k_B$=1.38 x $10^{-23}$ J/K
Mass of earth=597.42 ×$10^{22}$ kg, radius of earth = 6.378 × $10^6$ m

**PROBLEM 1**

A golfer is able to drive golf balls of mass 45 g a maximum range of 160 m. The golf ball stays in contact with the golf club while it travels 2.0 cm under constant acceleration. Ignore air resistance.

a. What is the magnitude of the golf ball's velocity just as its contact with the club ends? *(5 pts.)*

$$R = v_0^2/g \rightarrow v_0 = \sqrt{Rg} = \sqrt{160\text{m} \cdot 9.80\,\text{m/s}^2} =$$

**40. m/s**

b. What is the average force on the golf ball during contact? *(5 pts.)*

$$J = \Delta p = mv_0 = 0.045\text{kg} \cdot 40\frac{\text{m}}{\text{s}} = 1.8\text{Ns},\ \bar{v} = \frac{0 + 40\text{ms}^{-1}}{2} = 20\frac{\text{m}}{\text{s}},$$

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{0.02\text{m}}{20\text{ms}^{-1}} = 0.0010\text{s},\ \bar{F} = \frac{J}{\Delta t} = \frac{1.8\text{Ns}}{0.0010\text{s}} =$$

**1800 N**

c. What is the average power applied to the golf ball during contact? *(5 pts.)*

$$W = F \cdot d = 1800\text{N} \cdot 0.02\text{m} = 36\text{J or } W = K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.045\text{kg})(40\text{ms}^{-1})^2,\ P = \frac{W}{\Delta t} = \frac{36\text{J}}{0.0010\text{s}} =$$

**36 kW**

d. The velocity of the club head was 32 m/s before contact with the golf ball and 23 m/s after. What is the effective mass of the club head? *(5 pts.)*

$$m_c v_{cb} = m_c v_{ca} + mv_0 \Rightarrow m_c = \frac{v_0}{v_{cb} - v_{ca}} \cdot m = \frac{40\text{ms}^{-1}}{32\text{ms}^{-1} - 23\text{ms}^{-1}} \cdot 0.045\text{kg} =$$

**0.20 kg**

e. How much kinetic energy is lost in the collision between the golf ball and the club head? *(5 pts.)*

$$K_b = \tfrac{1}{2}m_c v_{cb}^2 = \tfrac{1}{2}(0.20\text{kg})(32\text{ms}^{-1})^2 = 102\text{J}$$

$$K_a = \tfrac{1}{2}m_c v_{ca}^2 + \tfrac{1}{2}m_0 v_0^2 = \tfrac{1}{2}(0.20\text{kg})(23\text{ms}^{-1})^2 + \tfrac{1}{2}(0.045\text{kg})(40\text{ms}^{-1})^2 = 53\text{J} + 36\text{J} = 89\text{J}$$

$$\Delta K = K_a - K_b = 102\text{J} - 89\text{J} =$$

**13 J**

**PROBLEM 2**

A flywheel with mass of 8.0 kg, a moment of inertia of 50.0 $kgm^2$ and a diameter of 1.8 m is rotating at 12 rev/s. It is stopped by two brake shoes that press against its edge with a force of 250 N each and a coefficient of friction of 0.60.

a. What is the centripetal acceleration of a point on the outside rim of the wheel before the brakes are applied? *(4 pts.)*

$$a_\perp = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r = \left(2\pi \times 12\frac{\text{rev}}{\text{s}}\right)^2 \times 0.90\text{m} = 5.1 \times 10^3 \text{ms}^{-2}$$

| 5100 $m/s^2$ |
|---|

b. What is the angular momentum of the flywheel before the brakes are applied? *(4 pts.)*

$L = I\omega = (50 \text{ kgm}^2)(2\pi \times 12 \text{ rev/s}) =$

| $3.8\times10^3 kgm^2s^{-1}$ |
|---|

c. What is the torque applied by the brake shoes on the flywheel? *(4 pts.)*

$\tau = 2r\mu F = 2(0.90\text{m})(0.60)(250\text{N}) =$

| 270 Nm |
|---|

d. What is the angular acceleration of the flywheel when the brakes are applied? *(4 pts.)*

$$\alpha = \tau/I = 270\text{Nm}/50\text{kgm}^2 =$$

| 5.4 rad/s |
|---|

e. How long does it take for the brakes to stop the flywheel? *(4 pts.)*

$$\omega_i = 2\pi \times 12\frac{\text{rev}}{\text{s}} = 75.4\frac{\text{rad}}{\text{s}},\ t = \frac{\omega_i - 0}{\alpha} = \frac{75.4\text{rads}^{-1}}{5.4\text{rads}^{-2}} =$$

| 14 s |
|---|

f. How much energy is dissipated in bringing the flywheel to rest? *(5 pts.)*

$$E = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(50\text{kgm}^{-2}\right)\left(75.4\text{rads}^{-1}\right)^2 =$$

| $1.4 \times 10^5$ J |
|---|

**PROBLEM 3**

Air (a diatomic ideal gas) at 21.0 °C and atmospheric pressure is drawn into a bicycle pump that has a cylinder with inner diameter of 2.70 cm and length 52.0 cm. The compression stroke adiabatically compresses the air, which reaches a gauge pressure of 708 kPa before entering the tire. The pump cylinder is thermally isolated from the outside but is in thermal contact with the inside air.

a. Determine the volume of the compressed air in the pump just before it enters the tire. *(5 pts.)*

For adiabatic compression, $P_1V_1^{\gamma}=P_2V_2^{\gamma}$, and for diatomic ideal gas $\gamma=7/5$, so

$V_2= V_1(P_1/P_2)^{(1/\gamma)} = (\pi(0.027/2\text{ m})^2(0.52\text{ m}))(1\text{ atm}/(1\text{ atm} + 708\text{ kPa}))^{5/7}$

$= (2.97729\times 10^{-4}\text{ m}^3)(1.013\times10^5/8.093\times10^5)^{5/7} =$

**$6.75\times10^{-5}\text{ m}^3$**

b. Determine the temperature of the compressed air in the pump just before it enters the tire. *(5 pts.)*

Find n: PV = nRT, so $n=P_1V_1/RT_1$

$=(1.013\times10^5)(2.97729\times 10^{-4})/(8.314)(273.1+21.0) = 0.01233$ mol. So

$T_2=P_2V_2/(nR)=(8.093\times10^5)(6.75\times10^{-5})/(0.01233)(8.314)=533\text{K}=$

**260 °C**

c. The pump is made of steel (density 7850 kg/m$^3$, specific heat C=0.456 kJ/kgK) and has an inner wall that is 2.00 mm thick. The pump is thermally isolated from the outside air, but is in thermal contact with the air inside it. Assume that 11.8 cm of the cylinder's length is allowed to come to thermal equilibrium with the air in the pump after completion of the compression stroke. What will be the increase in wall temperature? *(5 pts.)*

$(T_2-T_f)n_{air}C_{air}=(T_f-T_1)n_{steel}C_{steel}$, so $T_f=(T_2n_{air}C_{air}+T_1n_{steel}C_{steel})/(n_{steel}C_{steel}+n_{air}C_{air})$. Molar specific heat of air at constant volume is 5R/2 = (2.5)(8.314)=20.785 J/mol-K, and number of kg of steel is $(0.002\text{m}\times2\pi\times0.027/2\text{ m}\times0.118\text{ m})(7850\text{ kg/m}^3)=0.157$ kg. so

$$T_f=\frac{(533K)(0.01233mol)(20.785J/mol-K)+(294.1K)(0.157kg)(0.456kJ/kg-K)}{(0.01233mol)(20.785J/mol-K)+(0.157kg)(0.456kJ/kg-K)}=480.8K=208C.$$

So temperature change is 208-21=

**187 °C**

d. After many pump cycles, the tire is filled to a volume of 0.0135 m$^3$ and a gauge pressure of 708 kPa at 21.0 °C and sealed. How many moles of gas are on the tire? *(5 pts.)*

$n=PV/RT=((1.013+7.08)\times10^5\text{N/m}^2)(0.0135\text{ m}^3)/((8.314\text{ J/mol-K})(273.1+21.0\text{ K}))=$

**4.47 moles**

e. After a high speed ride, the tire air temperature rises to 32.7 °C and the interior volume of the tire increases by 1.55%. What is the gauge air pressure in the tire now? *(5 pts.)*

$P=nRT/V=(4.47\text{ mol})(8.314\text{ J/mol-K})(273.1+32.7\text{ K})/(1.0155\times0.0135\text{ m}^3)=828975$ Pa, so gauge pressure is 829000 Pa - 101350 Pa = 728000 Pa

**728 kPa**

**PROBLEM 4**

A container of volume 1.37 m$^3$ contains one mole of a mixture of 0.500 mole argon (Ar) gas and 0.500 mole nitrogen ($N_2$) gas in thermal equilibrium at 153 °C. (Mass of argon atom=$6.6335\times10^{-26}$ kg, mass of helium atom=$6.647\times10^{-27}$ kg)

a. What is the total translational kinetic energy, in Joules, of the mixture? *(5 pts.)*

$$KE_{trans} = \left(\frac{3}{2}RT\right) = \frac{3}{2}(8.3144 J mol^{-1} K^{-1})(153K + 273.1K) = 5314 \text{ J}$$

| 5310 J |
|---|

b. What is the root-mean-square speed for an argon atom? *(5 pts.)*

$$v_{RMS} = \sqrt{3k_B T / m} = \sqrt{(3)(5.88\times10^{-21} J)/(6.6335\times10^{-26} kg)} =$$

| 516 m/s |
|---|

c. What is the molar specific heat at constant volume of this gas mixture? *(5 pts.)*

$$C_P = \left(\frac{1}{2}mol\right)\left(\frac{3}{2}R\right) + \left(\frac{1}{2}mol\right)\left(\frac{5}{2}R\right) = 2R = (2)(8.3144) =$$

| 16.6 J/K |
|---|

d. What is the molar specific heat at constant pressure of this gas mixture? *(5 pts.)*

$$C_P = \left(\frac{1}{2}mol\right)\left(\frac{5}{2}R\right) + \left(\frac{1}{2}mol\right)\left(\frac{7}{2}R\right) = 3R = (3)(8.3144) =$$

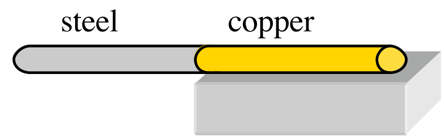
| 24.9 J/K |
|---|

e. The gas mixture expands adiabatically to a volume of 1.95 m$^3$. What is the pressure? *(5 pts.)*

$\gamma$ for this gas mixture is 3/2. For adiabatic expansion, $PV^{\gamma}$=constant. Initially V=1.37 m$^3$ and P=nRT/V=(1 mol)(8.3144 J mol$^{-1}$ K$^{-1}$)(426.1 K)/(1.37 m$^3$)=2585.35 Pa, so the constant is $(2585.35)(1.37)^{3/2}$=4145.72. So final P satisfies $P_f=(4145.72)/(1.95)^{3/2}$=1522 Pa.

| 1520 Pa |
|---|

## PROBLEM 5




A cylindrical steel rod (density 7850 kg/m$^3$, thermal expansion coefficient $13.0\times10^{-6}$/K, thermal conductivity $5.40\times10^4$ W/K, Young's modulus $2.00\times10^{11}$ N/m$^2$) and a cylindrical copper rod (density 8930 kg/m$^3$, thermal expansion coefficient $17.0\times10^{-6}$/K, thermal conductivity$4.01\times10^5$ W/K, Young's modulus $1.10\times10^{11}$ N/m$^2$), each of diameter 2.050 cm, are joined end to end and placed on an insulating block. At 22.5 °C, the steel part of the rod has a length of 0.778m, the left edge of the block is under the joint, and block is positioned so that the rod just stays on it without tipping off.

a. How long is the copper part of the rod? *(5 pts.)*

Rod stays on block if torque s can be balanced. Choose origin at pivot point:
Torque about joint=$M_{steel}\,L_{steel}/2-M_{copper}L_{copper}/2$+(possible torque from block, which cannot be less than zero). So, rod rotates about joint when $M_{steel}\,L_{steel}=M_{copper}L_{copper}$.
Since mass of each rod piece is $\rho AL$, $L_{copper} = L_{steel}(\rho_{steel}/\rho_{copper})^{1/2}$
= $(0.7780\text{m})(7850/8930)^{1/2}$ =

**0.729 m**

b. By how much does the length of the entire (compound) rod increase when the temperature is raised to 37.3 °C? *(5 pts.)*

Length increase $\Delta L = L\,\alpha\,\Delta T$, where L is the length, $\alpha$ is thermal expansion coefficient, and $\Delta T$ is temperature change. For steel, $\Delta L=(0.778\text{m})(13.0\times10^{-6}/\text{K})(37.3\text{C}-22.5\text{C})=0.000150$ m. For copper, $\Delta L=(0.729\text{m})(17.0\times10^{-6}/\text{K})(37.3\text{C}-22.5\text{C})=0.000183$ m.
So the total length increase is 0.000150 m+0.000183m =

**$3.33\times10^{-4}$ m**

c. Equal and opposite large inward forces of 189 N are applied to each end of the rod at 37.3 °C. What is the fractional change in the length of the compound rod induced by the stress?

For each metal, fractional change in length $\Delta L/L=(F/A)/Y$, so $\Delta L=LF/(AY)$.

$\Delta L_{steel}=(0.778\text{m})(189\text{ N})/(\pi(0.0205/2\text{ m})^2(2.00\times10^{11}\text{ N/m}^2))=2.23\times10^{-6}$
$\Delta L_{copper}=(0.729\text{m})(189\text{ N})/(\pi(0.0205/2\text{ m})^2(1.10\times10^{11}\text{ N/m}^2))=3.97\times10^{-6}$
$\Delta L/ L=(\Delta L_{steel}+\Delta L_{copper})/(L_{steel}+L_{copper})= (2.23+3.97)\times10^{-6}/(0.778+0.729)=$

**$4.11\times10^{-6}$**

For parts d and e, the rod is moved away to another place where it is wedged between two vertical walls. The left wall is maintained at 40.3 °C and the right wall at 22.5 °C, the steel part of the rod has a length of 0.778m and the copper part of the rod has a length of 0.722m. To a good approximation, all the heat flow between the walls occurs through the rod.

d. What is temperature at the joint? *(5 pts.)*

Heat flow is same across both rods, so $\kappa_{steel}(T_{left}-T_{joint})/L_{steel} = \kappa_{copper}(T_{joint}-T_{right})/L_{copper}$.
$\kappa_{steel}(T_{left}-T_{joint})/L_{steel} = \kappa_{copper}(T_{joint}-T_{right})/L_{copper}$, so
$T_{joint}=(\kappa_{steel}T_{left}/L_{steel}+\kappa_{steel}T_{right}/L_{steel})/(\kappa_{copper}/L_{copper}+\kappa_{steel}/L_{steel})=$

$$=\frac{5.40\times10^4(273.1+40.3)/0.778+4.01\times10^5(273.1+22.5)/0.722}{4.01\times10^5/0.722+5.40\times10^4/0.788}=298.0\text{K}$$

**24.9°C**

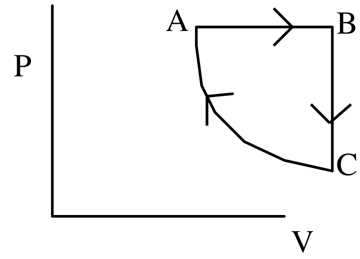
e. What is rate at which heat flows down the rod? *(5 pts.)*

$P = \kappa_{steel}(T_{left}-T_{joint})A/L_{steel} =(5.40\times10^4)(40.3-24.9)\pi(0.01025)^2/0.778 =$

**353 W**

**PROBLEM 6**

In the figure, the change in internal energy of one mole of a diatomic ideal gas that is taken from A to B is +790.0 J. The work done on the gas along the path ABC is –316.0 J. The volume at point C is 1.50 times that at A, and the path AC is an isothermal process.




a. How many Joules of energy must be added to the system by heat as it goes from *A* to *B*? *(5 pts.)*

$\Delta Q = \Delta E - \Delta W = (790.0\ J - (-316.0\ J)) =$

**1106 J**

b. How many Joules of heat are ejected into the surroundings on the leg BC? *(5 pts.)*

Internal energy at C is same as at A, so it changes by –790 J along BC leg. No work done on this leg, so heat ejected = $\Delta E$ =

**790 J**

c. How many Joules of heat are added to the gas along the leg CA? *(5 pts.)*

$\Delta Q = \Delta E - \Delta W$, and $\Delta E=0$. $\Delta W=\int P\,dV=nRT\int dV/V=nRT\ln(V_A/V_C) = R\ln(2/3)T$. Find T by noting $P_AV_A(V_B/V_A-1)=316$ J, so
$T_A=P_AV_A/(R)=(632)/(8.314)=76.0$ K. Since $T_B=3T_A/2$, $T_A=76$ K.
So $\Delta Q = (8.3144\ J/K)\ln(2/3)(76\ K) =$

**-256 J**

d. What is the change in entropy along the leg CA? *(5 pts.)*

$\Delta Q = T\Delta S$, so $\Delta S = \Delta Q/T$. Can find T as in c:
$T_A=P_AV_A/(R)=(632)/(8.314)=76.0$ K. So $\Delta S=(256\ J)/(76.0K)=$

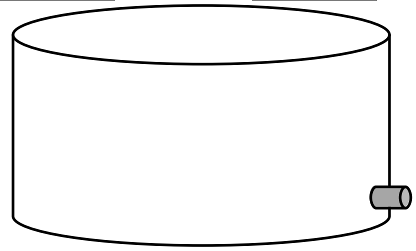
**3.37 J/K**

e. What is the efficiency ε of this engine? *(5 pts.)*
Efficiency is (work done by gas)/(heat input)

Heat input is 1106 J
Work done by gas on leg AB is 316 J
Work done on gas on leg CA is R ln(3/2)(76 K)=(8.314) ln(3/2)) (76) =256 J
So efficiency is (316-256)/1106 =

**0.0542**

## PROBLEM 7



A large circular tank with a diameter of 12.2 m and an open top contains water. The water can drain from the tank through a hose of diameter 6.80 cm. The hose ends with a nozzle of diameter 2.20 cm. A rubber stopper is inserted into the nozzle. The water level in the tank is kept 8.16 m above the nozzle.

a. Calculate the magnitude of the friction force exerted by the nozzle on the stopper. *(5 pts.)*

Pressure at nozzle is $\rho gh$, and force is (pressure×area). Force from water is balanced by friction force, so they have equal magnitudes.

So, friction force $=\rho ghA=(10^3\ kg/m^3)(9.80\ m/s^2)(8.16\ m)(\pi(0.011\ m)^2)=$ **30.4 N**

b. The stopper is removed. What mass of water flows from the nozzle in 0.21 s? *(5 pts.)*

Bernoulli: $P + \rho v^2/2+\rho gy=$constant. At the outlet the pressure is ambient, so $\rho v^2/2=8.00\times10^4$ Pa, and $v=(16\times10^4/10^3)^{1/2}=12.6466$ m/s. (Or, equivalently, use Torricelli's law $v= (2gh)^{1/2}$.) The mass flow in 0.21 s is
$\rho vAt=(10^3\ kg/m^3)(12.6466\ m/s)((\pi(0.011\ m)^2)(0.21\ s) =$ **1.01 kg**

c. The tank is placed on a rocket ship. Just after blastoff the rocket is at the earth's surface accelerating at 125 m/s$^2$. The water level in the tank is still kept constant at 8.16 m above the nozzle. What mass of water now flows from the nozzle in 0.21 s? *(5 pts.)*

The effect of acceleration is to increase the effective value of g:
$g_{eff} = g+a = (9.80\ m/s^2 + 125\ m/s^2) =134.8\ m/s^2$. The flow rate is proportional to $(g_{eff})^{1/2}$, so acceleration causes the amount of mass to increase by the factor $(g_{eff}/g)^{1/2}=(134.8/9.8)^{1/2}=3.709$.

So the mass of water that flows out in 0.21 s is (1.01 kg)(3.709) = **3.75 kg**

d. After the rocket has finished burning, the satellite is in a circular orbit around the earth with a period of two days. How far is the satellite from the earth's surface? *(5 pts.)*

For period T, gravitational acceleration $= v^2/R=(2\pi R/T)^2/R = 4\pi^2R/T^2$
$F=ma \Rightarrow GM_{earth}/R^2=4\pi^2R/T^2$, so $R=(GM_{earth}T^2/4\pi^2)^{1/3}$
$=((6.67 \times 10^{-11}\ Nm^2/kg^2)(597.42 \times10^{22}\ kg)(48\times3600\ s)^2/4\pi^2)^{1/3}$
$=6.705 \times 10^7$ m. Distance above earth $= (67.05-6.378) \times 10^6$ m= **$6.07 \times 10^7$ m**

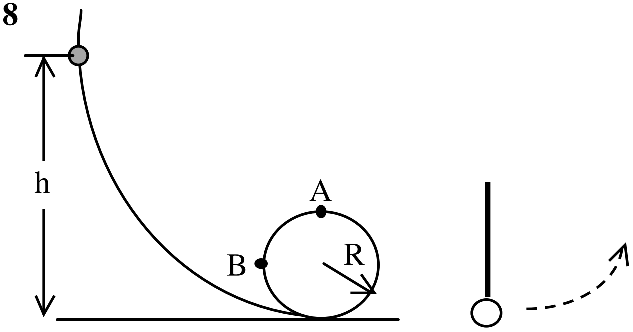
e. What is the magnitude of the acceleration of the satellite in the orbit in part d? *(5 pts.)*

$a=v^2/R=4\pi^2R/T^2=(4\pi^2)(6.07\times 10^7\ m)/(48\times3600\ s)^2 =$

**0.080 ms$^{-2}$**

**PROBLEM 8**

A bead slides without friction around a vertical loop-the-loop (see figure). The bead is released from a height $h = 3.31R$. The loop-the-loop radius is R=1.73m.




a. What is the bead's speed at point A? (5 pts.)

Change in potential energy is mg(3.31 R – 2 R)=(1.31)mgR=$mv^2/2$.
So $v=((2)(1.31)gR)^{1/2} = (2.62\times9.80\times1.73)^{1/2}$ =

**6.66 m/s**

b. What is the magnitude of the normal force on the bead at point A if its mass is 4.00 g? *(5 pts.)*

$mv^2/R = mg+N$, so $N=mv^2/R - mg=(0.004\text{ kg})((6.66\text{ m/s})^2/(1.73\text{ m}) - 9.8\text{ m/s}^2)$ =

**0.0634 N**

c. At point B, what is the direction of bead's acceleration (choose from up, down, right, left, up and right, up and left, down and right, and down and left)? *(5 pts.)*

Force from track is normal (horizontal), points to right to keep particle on circular path. Gravitational force points downward. Vector sum of these forces points

**down and right**

d. Upon leaving the loop-the-loop, the particle collides with the ball of a pendulum with mass 4.00 g hanging by a massless rod of length L=10.3 m. How long after this collision does the pendulum ball first reach its maximum height? *(5 pts.)*

Time to maximum height is 1/4 pendulum period
$= (1/4)(2\pi/(g/L)^{1/2}) = (1/2)(3.14159)/(9.8/10.3)^{1/2}$ =

**1.61 s**

e. What is the maximum height reached by the pendulum ball? *(5 pts.)*

Because masses are equal, just after collision the pendulum ball has the same velocity as the incoming bead. So pendulum rises so that it is at same height as bead is initially.
So h = 3.31*1.73 =

**5.73 m**