## Chapter Thirteen: Universal Gravitation

## SOLUTIONS TO PROBLEMS

**P13.9** 
$$a = \frac{MG}{(4R_E)^2} = \frac{9.80 \text{ m/s}^2}{16.0} = \boxed{0.613 \text{ m/s}^2}$$
 toward the Earth.

 $g = \frac{GM}{R^2} = \frac{G\rho\left(\frac{4\pi R^3}{3}\right)}{R^2} = \frac{4}{3}\pi G\rho R$ 

P13.10

If 
$$\frac{g_M}{g_E} = \frac{1}{6} = \frac{\frac{4\pi G\rho_M R_M}{3}}{\frac{4\pi G\rho_E R_E}{2}}$$

then 
$$\frac{\rho_M}{\rho_E} = \left(\frac{g_M}{g_E}\right) \left(\frac{R_E}{R_M}\right) = \left(\frac{1}{6}\right) (4) = \frac{2}{3}.$$

**P13.17** By Kepler's Third Law,  $T^2 = ka^3$  (*a* = semi-major axis)

For any object orbiting the Sun, with *T* in years and *a* in A.U., k = 1.00. Therefore, for Comet Halley

$$(75.6)^2 = (1.00) \left(\frac{0.570 + y}{2}\right)^3$$

The farthest distance the comet gets from the Sun is

$$-2a = x + y$$

x



$$y = 2(75.6)^{2/3} - 0.570 = 35.2$$
 A.U. (out around the orbit of Pluto)

**P13.39**  $E_{\text{tot}} = -\frac{GMm}{2r}$ 

$$\Delta E = \frac{GMm}{2} \left(\frac{1}{r_i} - \frac{1}{r_f}\right) = \frac{\left(6.67 \times 10^{-11}\right) \left(5.98 \times 10^{24}\right)}{2} \frac{10^3 \text{ kg}}{10^3 \text{ m}} \left(\frac{1}{6\ 370 + 100} - \frac{1}{6\ 370 + 200}\right)$$
$$\Delta E = 4.69 \times 10^8 \text{ J} = \boxed{469 \text{ MJ}}$$

\***P13.53** (a) Each bit of mass *dm* in the ring is at the same distance from the object at A. The separate contributions  $-\frac{Gmdm}{r}$  to the system energy add up to  $-\frac{GmM_{\text{ring}}}{r}$ . When the object is at A, this is

$$\frac{-6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 1\ 000 \text{ kg}\ 2.36 \times 10^{20} \text{ kg}}{\text{kg}^2 \sqrt{\left(1 \times 10^8 \text{ m}\right)^2 + \left(2 \times 10^8 \text{ m}\right)^2}} = \boxed{-7.04 \times 10^4 \text{ J}}.$$

(b) When the object is at the center of the ring, the potential energy is

$$\frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \text{ 1 000 kg } 2.36 \times 10^{20} \text{ kg}}{\text{kg}^2 \text{ 1} \times 10^8 \text{ m}} = \boxed{-1.57 \times 10^5 \text{ J}}$$

continued on next page

(c) Total energy of the object-ring system is conserved:

$$\begin{pmatrix} K + U_g \end{pmatrix}_A = \begin{pmatrix} K + U_g \end{pmatrix}_B 0 - 7.04 \times 10^4 \text{ J} = \frac{1}{2}1\ 000 \text{ kg}v_B^2 - 1.57 \times 10^5 \text{ J} v_B = \left(\frac{2 \times 8.70 \times 10^4 \text{ J}}{1\ 000 \text{ kg}}\right)^{1/2} = \boxed{13.2 \text{ m/s}}$$

P13.54 To approximate the height of the sulfur, set

$$\frac{mv^2}{2} = mg_{Io}h \qquad h = 70\ 000\ \text{m} \qquad g_{Io} = \frac{GM}{r^2} = 1.79\ \text{m/s}^2$$
$$v = \sqrt{2\ g_{Io}h}$$
$$v = \sqrt{2\ (1.79)(70\ 000)} \approx 500\ \text{m/s} \ (\text{over 1 000\ mi/h})$$

A more precise answer is given by

$$\frac{1}{2}mv^2 - \frac{GMm}{r_1} = -\frac{GMm}{r_2}$$

$$\frac{1}{2}v^{2} = (6.67 \times 10^{-11})(8.90 \times 10^{22})\left(\frac{1}{1.82 \times 10^{6}} - \frac{1}{1.89 \times 10^{6}}\right)$$

$$v = \boxed{492 \text{ m/s}}$$

**P13.62** (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum of the Earth is conserved;

$$mr_{a} v_{a} = mr_{p} v_{p} \text{ and } v_{a} = v_{p} \left(\frac{r_{p}}{r_{a}}\right) = \left(3.027 \times 10^{4} \text{ m/s}\right) \left(\frac{1.471}{1.521}\right) = \boxed{2.93 \times 10^{4} \text{ m/s}}$$
(b)  $K_{p} = \frac{1}{2} mv_{p}^{2} = \frac{1}{2} \left(5.98 \times 10^{24}\right) \left(3.027 \times 10^{4}\right)^{2} = \boxed{2.74 \times 10^{33} \text{ J}}$ 
 $U_{p} = -\frac{GmM}{r_{p}} = -\frac{\left(6.673 \times 10^{-11}\right) \left(5.98 \times 10^{24}\right) \left(1.99 \times 10^{30}\right)}{1.471 \times 10^{11}} = \boxed{-5.40 \times 10^{33} \text{ J}}$ 
(c) Using the same form as in part (b),  $K_{a} = \boxed{2.57 \times 10^{33} \text{ J}}$  and  $U_{a} = \boxed{-5.22 \times 10^{33} \text{ J}}$ .

Using the same form as in part (b),  $K_a = \boxed{2.57 \times 10^{33} \text{ J}}$  and  $U_a = \boxed{-5.22 \times 10^{33} \text{ J}}$ . Compare to find that  $K_p + U_p = \boxed{-2.66 \times 10^{33} \text{ J}}$  and  $K_a + U_a = \boxed{-2.65 \times 10^{33} \text{ J}}$ . They

agree.

## **Chapter Fourteen: Fluid Mechanics**

## SOLUTIONS TO PROBLEMS

P14.10 Suppose the "vacuum cleaner" functions as a high-vacuum pump. The air below the (a) brick will exert on it a lifting force

$$F = PA = 1.013 \times 10^5 \text{ Pa} \left[ \pi \left( 1.43 \times 10^{-2} \text{ m} \right)^2 \right] = 65.1 \text{ N}.$$

(b) The octopus can pull the bottom away from the top shell with a force that could be no larger than

$$F = PA = (P_0 + \rho gh)A = \left[1.013 \times 10^5 \text{ Pa} + (1\ 030 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(32.3 \text{ m})\right] \left[\pi (1.43 \times 10^{-2} \text{ m})^2\right]$$
$$F = \boxed{275 \text{ N}}$$

The pressure on the bottom due to the water is  $P_b = \rho g z = 1.96 \times 10^4$  Pa 14.12

So,
$$F_b = P_b A = \begin{bmatrix} 5.88 \times 10^6 \text{ N} \end{bmatrix}$$
On each end, $F = \overline{P}A = 9.80 \times 10^3 \text{ Pa}(20.0 \text{ m}^2) = \boxed{196 \text{ kN}}$ On the side, $F = \overline{P}A = 9.80 \times 10^3 \text{ Pa}(60.0 \text{ m}^2) = \boxed{588 \text{ kN}}$ 

**P14.20** Let *h* be the height of the water column added to the right side of the U-tube. Then when equilibrium is reached, the situation is as shown in the sketch at right. Now consider two points, *A* and *B* shown in the sketch, at the level of the water-mercury interface. By Pascal's Principle, the absolute pressure at *B* is the same as that at *A*. But,

$$P_A = P_0 + \rho_w g h + \rho_{\text{Hg}} g h_2 \text{ and}$$
$$P_B = P_0 + \rho_w g (h_1 + h + h_2).$$

Thus, from  $P_A = P_B$ ,  $\rho_w h_1 + \rho_w h + \rho_w h_2 = \rho_w h + \rho_{Hg} h_2$ , or

$$h_{\rm I} = \left[\frac{\rho_{\rm Hg}}{\rho_{w}} - 1\right] h_{\rm Z} = (13.6 - 1)(1.00 \text{ cm}) = \boxed{12.6 \text{ cm}}$$

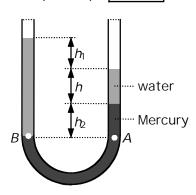


FIG. P14.20