## Chapter Thirteen: Universal Gravitation

## SOLUTIONS TO PROBLEMS

P13.9 $a=\frac{M G}{\left(4 R_{E}\right)^{2}}=\frac{9.80 \mathrm{~m} / \mathrm{s}^{2}}{16.0}=0.613 \mathrm{~m} / \mathrm{s}^{2}$ toward the Earth.

P13.10
$g=\frac{G M}{R^{2}}=\frac{G \rho\left(\frac{4 \pi R^{3}}{3}\right)}{R^{2}}=\frac{4}{3} \pi G \rho R$
If $\quad \frac{g_{M}}{g_{E}}=\frac{1}{6}=\frac{\frac{4 \pi G \rho_{M} R_{M}}{3}}{\frac{4 \pi G \rho_{E} R_{E}}{3}}$
then $\quad \frac{\rho_{M}}{\rho_{E}}=\left(\frac{g_{M}}{g_{E}}\right)\left(\frac{R_{E}}{R_{M}}\right)=\left(\frac{1}{6}\right)(4)=\frac{2}{3}$.
P13.17 By Kepler's Third Law, $T^{2}=k a^{3} \quad(a=$ semi-major axis)
For any object orbiting the Sun, with $T$ in years and $a$ in A.U., $k=1.00$. Therefore, for Comet Halley

$$
(75.6)^{2}=(1.00)\left(\frac{0.570+y}{2}\right)^{3}
$$

The farthest distance the comet gets from the Sun is


FIG. P13.17
$y=2(75.6)^{2 / 3}-0.570=35.2$ A.U. (out around the orbit of Pluto)
P13.39 $E_{\text {tot }}=-\frac{G M m}{2 r}$
$\Delta E=\frac{G M m}{2}\left(\frac{1}{r_{i}}-\frac{1}{r_{f}}\right)=\frac{\left(6.67 \times 10^{-11}\right)\left(5.98 \times 10^{24}\right)}{2} \frac{10^{3} \mathrm{~kg}}{10^{3} \mathrm{~m}}\left(\frac{1}{6370+100}-\frac{1}{6370+200}\right)$
$\Delta E=4.69 \times 10^{8} \mathrm{~J}=469 \mathrm{MJ}$
(a) Each bit of mass $d m$ in the ring is at the same distance from the object at A. The separate contributions $-\frac{G m d m}{r}$ to the system energy add up to $-\frac{G m M_{\text {ring }}}{r}$. When the object is at A, this is

$$
\frac{-6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} 1000 \mathrm{~kg} 2.36 \times 10^{20} \mathrm{~kg}}{\mathrm{~kg}^{2} \sqrt{\left(1 \times 10^{8} \mathrm{~m}\right)^{2}+\left(2 \times 10^{8} \mathrm{~m}\right)^{2}}}=-7.04 \times 10^{4} \mathrm{~J} .
$$

(b) When the object is at the center of the ring, the potential energy is

$$
-\frac{6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} 1000 \mathrm{~kg} \mathrm{2.36} \mathrm{\times 10}^{20} \mathrm{~kg}}{\mathrm{~kg}^{2} 1 \times 10^{8} \mathrm{~m}}=-1.57 \times 10^{5} \mathrm{~J} .
$$

continued on next page
(c) Total energy of the object-ring system is conserved:

$$
\begin{aligned}
& \left(K+U_{g}\right)_{A}=\left(K+U_{g}\right)_{B} \\
& 0-7.04 \times 10^{4} \mathrm{~J}=\frac{1}{2} 1000 \mathrm{~kg}_{B}^{2}-1.57 \times 10^{5} \mathrm{~J} \\
& v_{B}=\left(\frac{2 \times 8.70 \times 10^{4} \mathrm{~J}}{1000 \mathrm{~kg}}\right)^{1 / 2}=13.2 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

P13.54 To approximate the height of the sulfur, set

$$
\begin{aligned}
& \frac{m v^{2}}{2}=m g_{I o} h \quad \quad h=70000 \mathrm{~m} \quad g_{I o}=\frac{G M}{r^{2}}=1.79 \mathrm{~m} / \mathrm{s}^{2} \\
& v=\sqrt{2 g_{I o} h} \\
& v=\sqrt{2(1.79)(70000)} \approx 500 \mathrm{~m} / \mathrm{s}(\text { over } 1000 \mathrm{mi} / \mathrm{h})
\end{aligned}
$$

A more precise answer is given by
$\frac{1}{2} m v^{2}-\frac{G M m}{r_{1}}=-\frac{G M m}{r_{2}}$
$\frac{1}{2} v^{2}=\left(6.67 \times 10^{-11}\right)\left(8.90 \times 10^{22}\right)\left(\frac{1}{1.82 \times 10^{6}}-\frac{1}{1.89 \times 10^{6}}\right) \quad v=492 \mathrm{~m} / \mathrm{s}$
P13.62 (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum of the Earth is conserved;

$$
m r_{a} v_{a}=m r_{p} v_{p} \text { and } v_{a}=v_{p}\left(\frac{r_{p}}{r_{a}}\right)=\left(3.027 \times 10^{4} \mathrm{~m} / \mathrm{s}\right)\left(\frac{1.471}{1.521}\right)=2.93 \times 10^{4} \mathrm{~m} / \mathrm{s}
$$

(b) $\quad K_{p}=\frac{1}{2} m v_{p}^{2}=\frac{1}{2}\left(5.98 \times 10^{24}\right)\left(3.027 \times 10^{4}\right)^{2}=2.74 \times 10^{33} \mathrm{~J}$

$$
U_{p}=-\frac{G m M}{r_{p}}=-\frac{\left(6.673 \times 10^{-11}\right)\left(5.98 \times 10^{24}\right)\left(1.99 \times 10^{30}\right)}{1.471 \times 10^{11}}=-5.40 \times 10^{33} \mathrm{~J}
$$

(c) Using the same form as in part (b), $K_{a}=2.57 \times 10^{33} \mathrm{~J}$ and $U_{a}=-5.22 \times 10^{33} \mathrm{~J}$.

Compare to find that $K_{p}+U_{p}=-2.66 \times 10^{33} \mathrm{~J}$ and $K_{a}+U_{a}=-2.65 \times 10^{33} \mathrm{~J}$. They agree.

## Chapter Fourteen: Fluid Mechanics

## SOLUTIONS TO PROBLEMS

P14.10 (a) Suppose the "vacuum cleaner" functions as a high-vacuum pump. The air below the brick will exert on it a lifting force

$$
F=P A=1.013 \times 10^{5} \mathrm{~Pa}\left[\pi\left(1.43 \times 10^{-2} \mathrm{~m}\right)^{2}\right]=65.1 \mathrm{~N}
$$

(b) The octopus can pull the bottom away from the top shell with a force that could be no larger than

$$
\begin{aligned}
& F=P A=\left(P_{0}+\rho g h\right) A=\left[1.013 \times 10^{5} \mathrm{~Pa}+\left(1030 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(32.3 \mathrm{~m})\right]\left[\pi\left(1.43 \times 10^{-2} \mathrm{~m}\right)^{2}\right] \\
& F=275 \mathrm{~N}
\end{aligned}
$$

14.12 The pressure on the bottom due to the water is $\quad P_{b}=\rho g z=1.96 \times 10^{4} \mathrm{~Pa}$

So,
On each end,
On the side,

$$
\begin{aligned}
& F_{b}=P_{b} A=5.88 \times 10^{6} \mathrm{~N} \\
& F=\bar{P} A=9.80 \times 10^{3} \mathrm{~Pa}\left(20.0 \mathrm{~m}^{2}\right)=196 \mathrm{kN} \\
& F=\bar{P} A=9.80 \times 10^{3} \mathrm{~Pa}\left(60.0 \mathrm{~m}^{2}\right)=588 \mathrm{kN}
\end{aligned}
$$

P14.20 Let $h$ be the height of the water column added to the right side of the U-tube. Then when equilibrium is reached, the situation is as shown in the sketch at right. Now consider two points, $A$ and $B$ shown in the sketch, at the level of the water-mercury interface. By Pascal's Principle, the absolute pressure at $B$ is the same as that at $A$. But,

$$
\begin{aligned}
& P_{A}=P_{0}+\rho_{w} g h+\rho_{\mathrm{Hg}} g h_{2} \text { and } \\
& P_{B}=P_{0}+\rho_{w} g\left(h_{1}+h+h_{2}\right) .
\end{aligned}
$$

Thus, from $P_{A}=P_{B}, \rho_{w} h_{1}+\rho_{w} h+\rho_{w} h_{2}=\rho_{w} h+\rho_{\mathrm{Hg}} h_{2}$, or


FIG. P14.20

$$
h_{1}=\left[\frac{\rho_{\mathrm{Hg}}}{\rho_{w}}-1\right] h_{2}=(13.6-1)(1.00 \mathrm{~cm})=12.6 \mathrm{~cm} .
$$

