## Chapter Eleven: Angular Momentum

## SOLUTIONS TO PROBLEMS

P11.8
(a) $\quad \tau=\mathbf{r} \times \mathbf{F}=\left|\begin{array}{lll}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 3 & 0 \\ 3 & 2 & 0\end{array}\right|=\hat{\mathbf{i}}(0-0)-\hat{\mathbf{j}}(0-0)+\hat{\mathbf{k}}(2-9)=(-7.00 \mathrm{~N} \cdot \mathrm{~m}) \hat{\mathbf{k}}$
(b) The particle's position vector relative to the new axis is $1 \hat{\mathbf{i}}+3 \hat{\mathbf{j}}-6 \hat{\mathbf{j}}=1 \hat{\mathbf{i}}-3 \hat{\mathbf{j}}$.
$\tau=\left|\begin{array}{ccc}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -3 & 0 \\ 3 & 2 & 0\end{array}\right|=(11.0 \mathrm{~N} \cdot \mathrm{~m}) \hat{\mathbf{k}}$

P11.16 (a) The net torque on the counterweight-cord-spool system is:

$$
|\tau|=|\mathrm{r} \times \mathrm{F}|=8.00 \times 10^{-2} \mathrm{~m}(4.00 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=3.14 \mathrm{~N} \cdot \mathrm{~m} .
$$

(b) $\quad \mathbf{L}|=|\mathbf{r} \times m \mathbf{v}|+I \omega$

$$
|\mathbf{L}|=R m v+\frac{1}{2} M R^{2}\left(\frac{v}{R}\right)=R\left(m+\frac{M}{2}\right) v=(0.400 \mathrm{~kg} \cdot \mathrm{~m}) v
$$

(c) $\quad \tau=\frac{d L}{d t}=(0.400 \mathrm{~kg} \cdot \mathrm{~m}) a \quad a=\frac{3.14 \mathrm{~N} \cdot \mathrm{~m}}{0.400 \mathrm{~kg} \cdot \mathrm{~m}}=7.85 \mathrm{~m} / \mathrm{s}^{2}$

P11.18 Whether we think of the Earth's surface as curved or flat, we interpret the problem to mean that the plane's line of flight extended is precisely tangent to the mountain at its peak, and nearly parallel to the wheat field. Let the positive $x$ direction be eastward, positive $y$ be northward, and positive $z$ be vertically upward.
(a) $\quad \mathbf{r}=(4.30 \mathrm{~km}) \hat{\mathbf{k}}=\left(4.30 \times 10^{3} \mathrm{~m}\right) \hat{\mathbf{k}}$
$\mathbf{p}=m \mathbf{v}=12000 \mathrm{~kg}(-175 \hat{\mathbf{i}} \mathrm{~m} / \mathrm{s})=-2.10 \times 10^{6} \hat{\mathbf{i}} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$
$\mathbf{L}=\mathbf{r} \times \mathbf{p}=\left(4.30 \times 10^{3} \hat{\mathbf{k}} \mathrm{~m}\right) \times\left(-2.10 \times 10^{6} \hat{\mathbf{i}} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}\right)=\left(-9.03 \times 10^{9} \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}\right) \hat{\mathbf{j}}$
(b) No. $L=|\mathbf{r}| \mathbf{p} \mid \sin \theta=m v(r \sin \theta)$, and $r \sin \theta$ is the altitude of the plane. Therefore, $L=$ constant as the plane moves in level flight with constant velocity.
(c) Zero. The position vector from Pike's Peak to the plane is anti-parallel to the velocity of the plane. That is, it is directed along the same line and opposite in direction. Thus, $L=m v r \sin 180^{\circ}=0$.
*P11.26 $\sum F_{x}=m a_{x}: \quad+f_{s}=m a_{x}$
We must use the center of mass as the axis in

$$
\begin{array}{ll}
\sum \tau=I \alpha: \quad & F_{g}(0)-n(77.5 \mathrm{~cm})+f_{s}(88 \mathrm{~cm})=0 \\
\sum F_{y}=m a_{y}: & +n-F_{g}=0
\end{array}
$$

We combine the equations by substitution:


FIG. P11.26

$$
\begin{aligned}
& -m g(77.5 \mathrm{~cm})+m a_{x}(88 \mathrm{~cm})=0 \\
& a_{x}=\frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) 77.5 \mathrm{~cm}}{88 \mathrm{~cm}}=8.63 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

P11.29 $I_{i} \omega_{i}=I_{f} \omega_{f}: \quad\left(250 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)(10.0 \mathrm{rev} / \mathrm{min})=\left[250 \mathrm{~kg} \cdot \mathrm{~m}^{2}+25.0 \mathrm{~kg}(2.00 \mathrm{~m})^{2}\right] \omega_{2}$

$$
\omega_{2}=7.14 \mathrm{rev} / \mathrm{min}
$$

