ALL LABS + DISCUSSION Sections MEET THIS WEEK
HW DUE 1st Disc. Sec.
Quiz in 2nd Disc. Sec.

Average Velocity

$X = a + bt^2$

$a = 20\,cm,\ b = 4\,cm/s^2$

Displacement $t_1 = 2\,s,\ t_2 = 5\,s$

$t_1 = 2\,s:\ X_1 = 20\,cm + \left(4\,\frac{cm}{s^2}\right)(2\,s)^2 = 36\,cm$

$t_2 = 5\,s:\ X_2 = 20\,cm + \left(4\,\frac{cm}{s^2}\right)(5\,s)^2 = 120\,cm$

$X_2 - X_1 = 120\,cm - 36\,cm = 84\,cm$

$\bar{V} = \frac{\Delta X}{\Delta t} = \frac{X_2 - X_1}{t_2 - t_1} = \frac{84\,cm}{3\,s} = 28\,\frac{cm}{s}$

Instantaneous Velocity

$X = 20\,cm + \left(4\,\frac{cm}{s^2}\right)t^2$

$\frac{d}{dt}(t^n) = n\,t^{n-1}$:

$V = \frac{dX}{dt} = \frac{d}{dt}\left(20\,cm + 4\,\frac{cm}{s^2}t^2\right)$

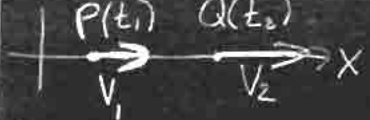
$V = 2\left(4\,\frac{cm}{s^2}\right)t$

Find V at $t = 2\,s$

$V = 2\left(4\,\frac{cm}{s^2}\right)2\,s = 16\,\frac{cm}{s}$

Average + Instantaneous Acceleration:

Velocity changes with time

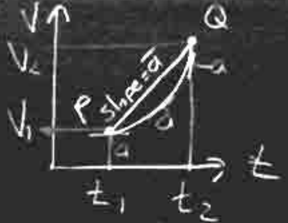


$$\bar{a} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta V}{\Delta t}$$

= Average Acceleration

Instant acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} \Rightarrow a = \frac{dV}{dt}$$



a = slope of tangent at a particular point

$\bar{a}$ = Slope of line between P and Q

$$a = \frac{dV}{dt} = \frac{d}{dt}\left(\frac{dX}{dt}\right) = \frac{d^2X}{dt^2}$$

Velocity as function of position

$$a = \frac{dV}{dt} = \frac{dV}{dX}\frac{dX}{dt} = V\frac{dV}{dX}$$

at t = 2 sec

$$V = m + nt^2 ; \quad m = 10\frac{cm}{s} \quad n = 2\frac{cm}{s^3}$$

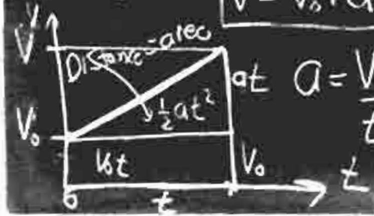
$$a = \frac{dV}{dt} = \frac{d}{dt}\left(10\frac{cm}{s} + \left(2\frac{cm}{s^3}\right)t^2\right)$$

$$= 2\left(2\frac{cm}{s^3}\right)t = 2\left(2\frac{cm}{s^3}\right)(2s) = 8\,cm/s^2$$

Special Case. Constant Acceleration

Velocity changes at a constant Rate. V changes ΔV in 1 sec, $t\Delta V$ in t secs. Let $\Delta V = a$

Start at $V = V_0$ at $t=0$, then after t secs: $V = V_0 + at$



$$a = \frac{V - V_0}{t - 0}$$

$V = V_0 + at$ *

Const. Accel

$V = V_0$ at $t = 0$

Instantaneous a = average $\bar{a}$

Average Velocity

$$\bar{V} = \frac{X_2 - X_1}{t_2 - t_1}$$

$t_1 = 0$, $t_2 = t$

$X_1 = X_0$, $X_2 = X$

$X - X_0 = \bar{V}t$

Find $\bar{V}$ in terms of a:

Average 3 Velocities: $0, \frac{t}{2}, t$

$t = 0$, $V = V_0$

$t = t/2$, $V_{1/2} = V_0 + at/2$

$t = t$, $V_1 = V_0 + at$

$$\bar{V} = \frac{1}{3}(V_0 + V_{1/2} + V_1) =$$

$$\frac{1}{3}\left(V_0 + V_0 + a\frac{t}{2} + V_0 + at\right) = \frac{3V_0 + \frac{3at}{2}}{3}$$

$\bar{V} = V_0 + \frac{1}{2}at$ true for any n times

Initial Velocity: $V_i = V_0$

final Velocity: $V_f = V_0 + at$

avg. Velocity: $\bar{V} = V_0 + \frac{1}{2}at$

$X - X_0 = \bar{V} t$

$= (V_0 + \frac{1}{2}at)t$

$$X - X_0 = V_0 t + \frac{1}{2}at^2 \quad *$$

Suppose don't know t:

$V = V_0 + at \Rightarrow t = \frac{V - V_0}{a}$

$X - X_0 = \left(V_0\left(\frac{V-V_0}{a}\right) + \frac{1}{2}a\left(\frac{V-V_0}{a}\right)^2\right)$

$\rightarrow X - X_0 = \left(V_0 + \frac{1}{2}(V - V_0)\right)\frac{V - V_0}{a}$

$= \left(\frac{V_0 + V}{2}\right)\left(\frac{V - V_0}{a}\right) = \frac{V^2 - V_0^2}{2a} = X - X_0$

$$V^2 = V_0^2 + 2a(X - X_0) \quad *$$

Don't know a:

$V = V_0 + at \rightarrow a = \frac{V - V_0}{t}$

$X - X_0 = V_0 t + \frac{1}{2}at^2$

$= V_0 t + \frac{1}{2}\left(\frac{V - V_0}{t}\right)t^2$

$\rightarrow V_0 t + \frac{1}{2}Vt - \frac{1}{2}V_0 t$

$$X - X_0 = \frac{1}{2}(V + V_0)t \quad *$$

Don't know V_0:

$V_0 = V - at$

$X - X_0 = \frac{1}{2}(V + V_0)t$

$= \frac{1}{2}(V + V - at)t$

$= \frac{1}{2}(2V - at)t$

$$X - X_0 = Vt - \frac{1}{2}at^2 \quad **$$

only true for const accel

Non-Constant accel zone:

area = displacement

$$V_2 - V_1 = \int_{V_1}^{V_2} dV = \int_{t_1}^{t_2} a\,dt$$

Freely Falling Bodies

(regardless of size, weight)

at same point on earth's surface

In absence of air resistance, fall at same rate = constant acceleration

$$a = -g, \quad g = 32\,\frac{ft}{s^2}$$

$$g = 9.8\,m/s^2$$

Vertical Coordinate

$+y \uparrow \quad g \downarrow$

$$V = V_0 + at \rightarrow V = V_0 - gt$$

$$X - X_0 = V_0 t + \frac{1}{2} a t^2 \rightarrow$$

$$Y - Y_0 = V_0 t - \frac{1}{2} g t^2$$

$$V^2 = V_0^2 + 2a(X - X_0) \rightarrow$$

$$V^2 = V_0^2 - 2g(Y - Y_0)$$

$$X - X_0 = \frac{1}{2}(V_0 + V)t \rightarrow$$

$$Y - Y_0 = \frac{1}{2}(V_0 + V)t$$

$$X - X_0 = Vt - \frac{1}{2} a t^2 \rightarrow$$

$$Y - Y_0 = Vt + \frac{1}{2} g t^2$$

Throw object in air.
Has speed of +10 m/s
at half its max height
How high does it go?

Solve: $t = 0$ at 1st piece of data

Start (half height): $V_0 = 10\,m/s$

$a = -g = -9.8\,m/s^2$

End (full height): $V = 0$

$a = -g = -9.8\,m/s^2$

Y_0: start, Y = end

$$Y - Y_0 = \frac{1}{2} h_{max}$$

$$V^2 = V_0^2 + 2a(Y - Y_0)$$

$$Y - Y_0 = \frac{V^2 - V_0^2}{2a}$$

$$= \frac{0 - (10\,m/s)^2}{2(-9.8\,m/s^2)} = +5.10\,m$$

$$Y - Y_0 = \frac{1}{2} h_{max} \Rightarrow h_{max} = 2(5.10\,m)$$

$$= 10.2\,m$$