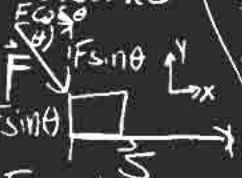
Work

Work = Force x distance

$W = \vec{F} \cdot \vec{d}$



$S_y = 0,\ F_y = F\sin\theta$

$S_x = S,\ F_x = F\cos\theta \quad (S = |\vec{S}|)$

$W = \vec{F}\cdot\vec{S} = F_x S_x + F_y \cancel{S_y}^{\,0}$

$= SF\cos\theta$

Force in same direction as displ: $W > 0$

F opposite to displ $\Rightarrow W < 0$

F perpendicular to displ $\Rightarrow W = 0$

$\vec{F} \perp \vec{S} \Rightarrow \vec{F}\cdot\vec{S} = 0 = W$

Units of Work = Joules

1 Joule = 1 Newton-meter.

Push 40 kg block on level floor

5.0 m at angle 30° below horiz

Coeff kinetic Friction = .25

$y: N - mg - F\sin\theta = 0$

$N = mg + F\sin\theta$

Travel at constant speed

$X: F\cos\theta - F_k = 0$

$F\cos\theta = F_k = \mu_k N$

$= \mu_k(mg + F\sin\theta)$

$F\cos\theta = \mu_k mg + \mu_k F\sin\theta$

$F(\cos\theta - \mu_k\sin\theta) = \mu_k mg$

$$F = \frac{\mu_k m g}{\cos\theta - \mu_k \sin\theta} \qquad \theta = 30°$$

$$= \frac{(.25)(40\,kg)(9.8\,\frac{m}{s^2})}{.866 - (.25)(.5)}$$

$= 132\,N = F$

Force in direction of displacement $= F\cos\theta$

$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$

$= (132N)(5.0m)(.866) = 572\,J$

Work by Force = by Friction

Work done by Varying force

Let force be constant $\vec{F}_i$ for small displacements $\Delta\vec{S}_i$

$W = \sum \vec{F}_i \cdot \Delta\vec{S}_i$

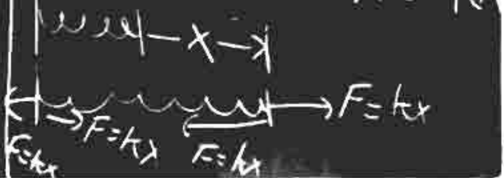
Let displ $\Delta S_i \to$ smaller

$$W = \lim_{\Delta x \to 0} \sum \vec{F}(x)\cdot\Delta\vec{x}$$

$$W = \int_{S_i}^{S_f} \vec{F}(x)\cdot d\vec{x}$$

Spring: Hooke's Law

extend distance $X \to F = -kX$



$$W = \frac{1}{2}x\,kx = \frac{1}{2}kx^2$$

Don't start at 0

Stretch from $x_1 \to x_2$

$$W = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

Work done by pulling agent

$$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

$$W = \int_{x_1}^{x_2} F(x)\,dx = \int_{x_1}^{x_2}(-kx)\,dx = -k\int_{x_1}^{x_2} x\,dx = -k\,\frac{x^2}{2}\Big|_{x_1}^{x_2} = -\frac{k}{2}(x_2^2 - x_1^2)$$

$$= \frac{k}{2}(x_1^2 - x_2^2) = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

Work and Kinetic Energy

mass moving under influence of $\vec{F}$:

$\vec{F} = m\vec{a}$, speed increases $v_1 \to v_2$

While undergoes displacement s

$$v^2 = v_0^2 + 2a(x - x_0) \Rightarrow v_2^2 = v_1^2 + 2as$$

$$a = \frac{v_2^2 - v_1^2}{2s} \Rightarrow F = m\left(\frac{v_2^2 - v_1^2}{2s}\right)$$

$$W = F\cdot s = m\left(\frac{v_2^2 - v_1^2}{2s}\right)s = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Kinetic Energy of Motion = K

$$\boxed{K = \frac{1}{2} m V^2}$$

Work of a resultant external force on a body = change in its kinetic energy

$$W = K_2 - K_1 = \Delta K$$

$$K_1 = \frac{1}{2} m v_1^2$$

Positive Work, K increases

Negative Work, K decreases

Zero Work ⇒ K unchanged

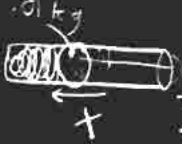
Example: spring gun

Spring Constant $500 \frac{N}{m}$

Compress 0.05 m

Insert ball of mass .01 kg

spring released → exit speed?



$$\Delta W = \Delta K$$

$$\frac{1}{2} k x^2 = \frac{1}{2} m v^2$$

$$V^2 = \frac{k x^2}{m} = \frac{\left(500 \frac{N}{m}\right)(.05 m)^2}{.01 kg}$$

$$= 125 \frac{N \cdot m}{kg} = 125 \frac{kg\, m s^{-2}\, m}{kg} = 125 m^2/s^2$$

$$V = \sqrt{125 \frac{m^2}{s^2}} = 11.2 m/s$$

Average Power $= \bar{P} = \frac{\text{Work done}}{\text{time interval}}$

$$\boxed{\bar{P} = \frac{\Delta W}{\Delta t}}$$

Instantaneous Power

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \boxed{\frac{dW}{dt} = P}$$

Units $= \frac{1 \text{ Joule}}{\text{sec}} = 1$ Watt

$kW = 10^3 W$, $MW = 10^6 W$

$1 hp = 746 W = \frac{3}{4} kW$

Unit of Energy = kilowatt-hour

Work done in 1 hour by agent working at rate of 1 kW

$1 kWh = 3600 \text{ sec} \times 1000 \frac{J}{\text{sec}} = 3.6 \times 10^6 J$

$= 3.6 MJ = 1 kWh$

MG+E charges 8 ¢ per kWh = 2 ¢ per MJ

Constant Force F applied to body moving at V undergoes displacement X

$W = F \cdot X$

$$\bar{P} = \frac{W}{t} = \frac{F \cdot X}{t} = F \cdot \frac{X}{t} = F \cdot \bar{V}$$

Since $\bar{V} = X/t \Rightarrow \boxed{\bar{P} = F \cdot \bar{V}}$

Force constant, time short

Inst. Power

$$\boxed{P = \vec{F} \cdot \vec{V}}$$

$$P = F\left(\frac{\Delta X}{\Delta t}\right)$$

Force constant

$$P = \frac{dW}{dt} = \frac{d}{dt}(\vec{F} \cdot \vec{X}).$$

$$\vec{P} = \vec{F}\frac{d\vec{x}}{dt} = \vec{F}\cdot\vec{V}$$

$$\boxed{\vec{P} = \vec{F}\cdot\vec{V}}$$

Find Force required to keep body in motion for a certain power

$$F = \frac{P}{V}$$