Rotation

Rigid Body rotating around fixed axis O

Draw reference point P ⟹ defines angle θ b/w OP and x axis

Units of θ = radians

1 radian is angle subtended by arc of length = radius of circle.

$2\pi r = 360°$

$r = 360/2\pi$

$= 57.3°$

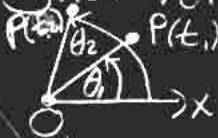
2π radians = 360°. For any angle θ subtended by arc s

$\theta = s/R$ = Angular coordinate

R = radius

Angular Velocity

Rotating point P.



Avg angular velocity

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

Inst. Angular Velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{d\theta}{dt}$$

Units = $\frac{\text{radians}}{\text{sec}}$

$\boxed{1 \text{ rev/sec} = 2\pi \text{ rad/sec}}$

Average Angular Acceleration

have ω_1 at t_1, ω_2 at t_2 ⇒

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

Inst. Angular Acceleration

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \Rightarrow \alpha = \frac{d\omega}{dt}$$

Units are rad/sec^2

Rotation with constant angular accel.

Start with ω_0 at $t=0$ at θ_0

at time t later have ω at θ

$$\alpha = \frac{\omega - \omega_0}{t - 0} \Rightarrow \boxed{\omega = \omega_0 + \alpha t}$$

(avg = inst for constant)

$$\bar{\omega} = \frac{\omega_0 + \omega}{2}, \quad \theta = \theta_0 + \bar{\omega}\, t \Rightarrow$$

$$\boxed{\theta = \theta_0 + \left(\frac{\omega + \omega_0}{2}\right) t}$$

Substitute $\omega = \omega_0 + \alpha t$

$$\theta = \theta_0 + \left(\frac{\omega_0 + \alpha t + \omega_0}{2}\right) t \Rightarrow$$

$$\boxed{\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2}$$

$$\omega = \omega_0 + \alpha t \Rightarrow t = \frac{\omega - \omega_0}{\alpha}$$

$$\theta = \theta_0 + \left(\frac{\omega + \omega_0}{2}\right)\left(\frac{\omega - \omega_0}{\alpha}\right)$$

$$\theta_0 + \frac{\omega^2 - \omega_0^2}{2\alpha} = \theta$$

$$\boxed{\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)}$$

like linear case

with $x \to \theta$, $v \to \omega$, $a \to \alpha$

Relation b/w linear + rotation, for fixed radius r.

Point P travelled $s = r\theta$

small $\theta = \Delta\theta$

$\Delta s = r\Delta\theta \Rightarrow \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$

$\frac{\Delta s}{\Delta t}$ = average speed during Δt

$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow \boxed{V = r\omega}$

If angular velocity only changes in magnitude by $\Delta\omega$ then ΔV changes by $\Delta V = r\Delta\omega \Rightarrow$

$$\frac{\Delta V}{\Delta t} = r\frac{\Delta\omega}{\Delta t} \Rightarrow \frac{dV}{dt} = r\frac{d\omega}{dt}$$

ΔV only changes in magnitude. $a_{\parallel} = \frac{\Delta V}{\Delta t} = \frac{dV}{dt}$

$\alpha = \frac{d\omega}{dt} = \frac{\Delta\omega}{\Delta t}$

$a_{\parallel}$ = acceleration parallel to velocity vector $\Rightarrow$ changes magnitude

$\underbrace{\boxed{a_{\parallel} = r\alpha}}_{\text{tangential}}$

But for radial acceleration:

$$\boxed{a_{\perp} = \frac{V^2}{r}} = \frac{(\omega r)^2}{r} = \underbrace{\boxed{\omega^2 r = a_{\perp}}}_{\text{radial}}$$

Kinetic Energy

$V = r\omega$ for a mass m at radius r moving at angular velocity ω:

$$\frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2$$

But we have a body made up of many points of mass m_i at radius r_i

$$K = \sum_i \frac{1}{2} m_i r_i^2 \omega^2$$

all points have same ω

$$K = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

Define $I = \sum_i m_i r_i^2$

I = Moment of Inertia

$$K = \frac{1}{2} I\omega^2 \Leftrightarrow \frac{1}{2}mV^2$$

Find I for a Solid body

divide into small Volumes dV

distance r from axis

with mass $\Delta m \to dm$

$$I = \lim_{\Delta m \to 0} \sum r^2 \Delta m = \int r^2 dm$$

If body uniform with density $\rho = \frac{dm}{dV}$

$$I = \int r^2 \rho \, dV = \rho \int r^2 dV$$

Non uniform body: $\rho(r)$

$$I = \int r^2 \rho(r) \, dV$$

Moment of Inertia is wrt a specific axis

(in Calculation of I, r_i depends axis location)

Given moment of Inertia through center of mass

$I_{cm} \Rightarrow$ Then I_p = I through a parallel axis a distance d away $I_p = I_{cm} + Md^2$

d = dist btw axis thru center of mass and new axis P

M = mass of body

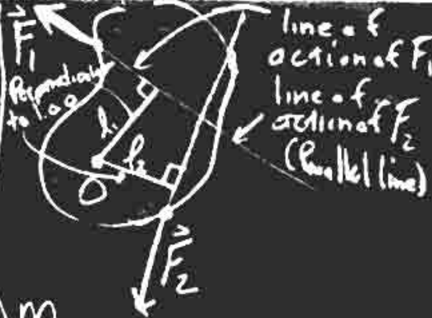
Parallel Axis Theorem

$$I_P = I_{cm} + Md^2$$

Work and Power

Analog of force for rotations

Body on axis with 2 Forces acting on it



Moment arm = Perpendicular from axis to line of action of a force

Torque $\tau = F \cdot l$

$$\tau_1 = F_1 l_1$$

$$\tau_2 = F_2 l_2$$