Phys 201

Exam 2
Mon 10/25 5:45-6:45pm
272 Bascom

Review Sessions
1300 Sterling

Friday 10/22 5-7pm
Sunday 10/24 2-4pm

Torque

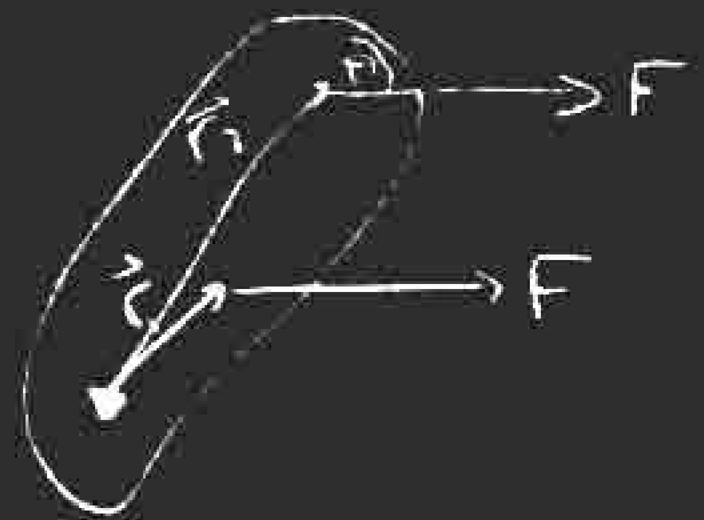
$[\tau = I\alpha]$

A given force is more successful at causing a change in rotational motion if it is applied farther away from the axis

[e.g. wrench, door, seesaw]

tendency of a force to rotate an object about some axis is measured by torque τ

torque is vector
[here will fix axis]



Define torque τ from force $\vec{F}$ [magnitude]

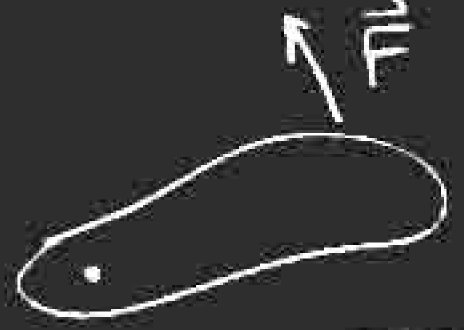
$$\tau = r F_\perp = r F \sin\theta$$

↑ distance to rotation axis

$F_\perp$ is component of $\vec{F}$ $\perp$ to $\vec{r}$

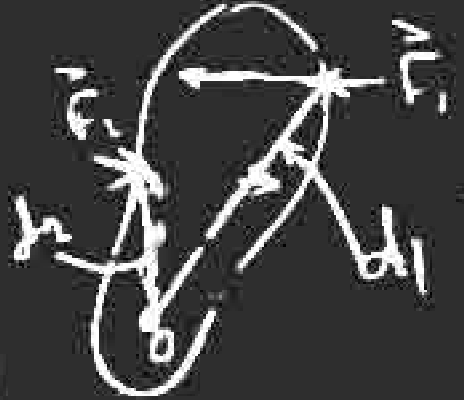
Sign convention:

$\tau > 0$ if induced rotation tendency is counterclockwise



$\tau > 0$

If 2 or more forces.



$$\tau = F_1^\perp d_1 + F_2^\perp d_2$$

Torque is not a force!

Relationship between torque and angular acceleration

$$\tau = I\alpha$$

Show $\tau = I\alpha$ for particle in circular orbit

$\vec{F} = m\vec{a}$

$F_t = m a_t$

torque $= RF_t$

$a_t = R\alpha$

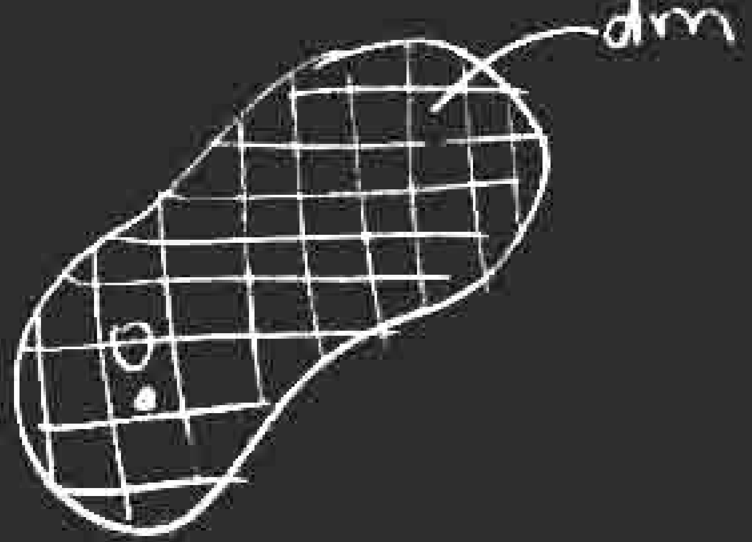
$\tau = RF_t = R(m)(R\alpha)$

$\tau = \underbrace{mR^2}_{}\alpha$

so $\tau = I\alpha$ for single particle in circular orbit

---

for rigid object of arbitrary shape rotating about fixed axis,



external force acting on dm has tangential component

$dF_t = dm\, a_t + [\text{forces from rest of body}]$

$d\tau = r\, dF_t = r\, dm\, a_t + [\ ]$

$\tau = \sum d\tau = \sum r\, dm\, a_t + 0$

So,

$$\tau = \sum d\tau = \sum dm\, \alpha r^2$$
$$= \alpha \sum dm\, r^2$$
$$= \alpha I$$

Work, Power, and Energy in Rotational motion

Work done by force $\vec{F}$ on body when it rotates through $d\theta$

$$dW = (\vec{\text{force}}) \cdot (\vec{\text{displacement}})$$

$$(F_\perp)(R)$$

$\perp$ to R

$$\underbrace{F \sin\varphi \; R}\, d\theta$$

$$\tau\, d\theta$$

So work $\boxed{dW = \tau\, d\theta}$

$$\text{Power} = \frac{dW}{dt} = \tau \frac{d\theta}{dt}$$

$$\boxed{P = \tau \omega}$$

Work-Kinetic Energy theorem

rotational work done by external forces

= change in rotational KE

prove work-KE thm
for rotational motion:

$$\sum \tau = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta}\frac{d\theta}{dt} = I\frac{d\omega}{d\theta}\omega$$

use

$$\sum \tau \, d\theta = I\frac{d\omega}{d\theta}\omega \, d\theta = I\omega \, d\omega$$

$$\parallel$$

$$dW = I\omega \, d\omega$$

integrate

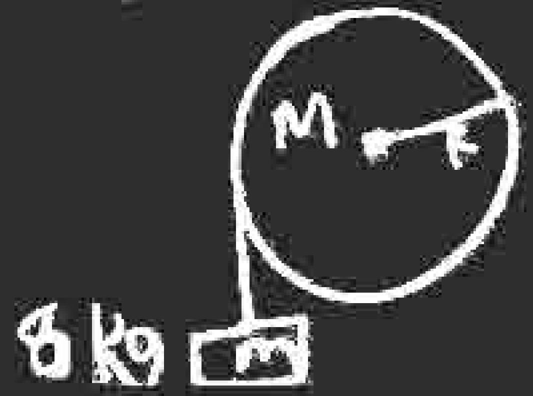
$$W = \int_{\omega_I}^{\omega_F} I\omega \, d\omega = \frac{1}{2}I\omega_F^2 - \frac{1}{2}I\omega_I^2$$

In general, have both translational & rotational KE

and net work = change in total KE

$$\sum_i \frac{1}{2}m_i v_i^2 + \sum_i \frac{1}{2}I_i\omega_i^2$$

Simple example



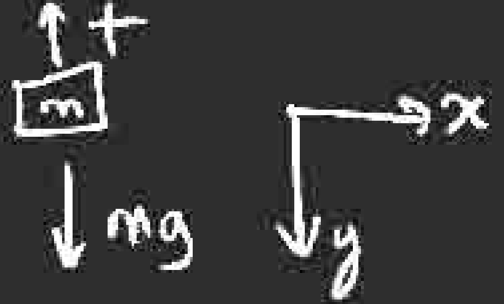
$M = 15$ kg

$(I = \frac{1}{2}MR^2)$

$R = 0.15$ m

Start from rest
what is acceleration of m?

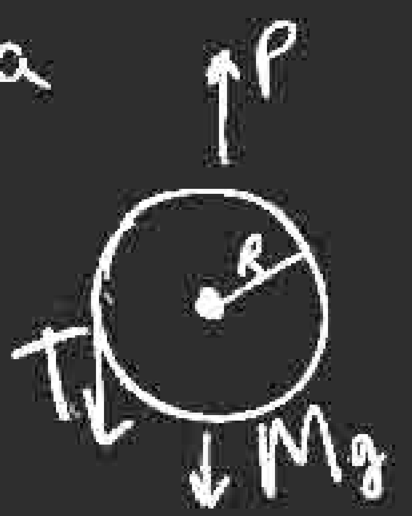
$\vec{F} = m\vec{a}$ for m



$mg - T = ma$

Cylinder:

$\tau = I\alpha$



P and Mg contribute no torque

$\tau = TR$

$\tau = I\alpha \Rightarrow TR = \left(\frac{1}{2}MR^2\right)\alpha$

$T = \frac{1}{2}MR\alpha$

need to relate a and α

$a_t = a_{block} = R\alpha$

of rope around cylinder

$\Rightarrow T = \frac{1}{2}Ma$

$[mg - T = ma]$

$T = m(g-a)$

$m(g-a) = \frac{1}{2}Ma$

$mg = \left(\frac{1}{2}M + m\right)a$

$a = \frac{mg}{\left(\frac{1}{2}M + m\right)} = \frac{(8\,kg)(9.8\,m/s^2)}{\left(\frac{1}{2}(15\,kg) + (8\,kg)\right)} = 5.06\ m/s^2$