Phys 201

No office hour today (11/9)

Damped Oscillations

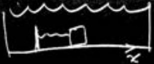
so far all our systems had no friction

- oscillate forever

"real" systems are damped.

consider "linear damping" where retarding force $\vec{R} = -b\vec{v}$ [b = damping coef]

(block in viscous fluid)



$$\Sigma F_x = -kx - bv_x = ma_x$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + \omega_0^2 x = 0$$

$$\beta = b/m, \quad \omega_0 = \sqrt{k/m}$$

solution is (if $\frac{b}{2m} < \sqrt{\frac{k}{m}}$)

$$x = Ae^{-\frac{b}{2m}t}\cos(\omega t + \varphi)$$

$$\omega = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}$$

frequency $\omega = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}$
decay rate = $\gamma/2$
(verify by substituting in)

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if $\left(\frac{b}{2m}\right)^2 \geq \frac{k}{m}$

$x = Ae^{-\Gamma t}$ no oscillations

when $\left(\frac{b}{2m}\right)^2 = \frac{k}{m}$ $\Gamma = -\frac{b}{2m}t$

↑

"critical damping"

Forced oscillations

subject damped oscillator to oscillatory forcing $F = F_0 \cos \omega t$

$$\Sigma F = ma \Rightarrow F_0 \cos \omega t - b\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

Steady state solution is

$$x = A\cos(\omega t + \delta)$$

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}} \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

$$\delta = \tan^{-1} \frac{\omega b/m}{\omega_0^2 - \omega^2}$$

1) entire system oscillates at drive frequency

2) amplitude is maximum at resonant frequency $\omega = \omega_0$

3) smaller damping ⇒ larger max amplitude

At resonance ($\omega = \omega_0$) applied force is in phase with velocity $\vec{F} \cdot \vec{v}$ is maximized

Ch 13. Newton's law of gravitation

Gravitational force between 2 point masses m_1 and m_2 separated by distance r is

$$F_g = G \frac{m_1 m_2}{r^2}$$

directed so that particles are attracted to each other

G = universal gravitational constant

$$G = 6.673 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$$

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12}$$

Grav. force exerted by Spherically symmetric mass is same (outside the mass) as if all mass were concentrated at center

So particle of mass m near earth's surface,

magnitude of grav. force $F_g = \frac{G M_E m}{R_E^2}$

M_E = earth's mass
R_E = earth's radius

$g = 9.80\ m/s^2$

$R_E = 6370\ km = 6.37 \times 10^6\ m$

$$m_E = \frac{R_E^2 g}{G} = \frac{(6.37 \times 10^6\ m)^2 (9.8\ m/s^2)}{6.67 \times 10^{-11}\ Nm^2\ kg^{-2}}$$

$$= 5.96 \times 10^{24}\ kg$$

Check: look at moon.

gravitational acceleration from earth's gravity $= \frac{G M_E}{r_m^2}$ ← earth-moon separation

$$= \frac{G M_E}{R_E^2}\left(\frac{r_E^2}{r_m^2}\right) = g\left(\frac{r_E^2}{r_m^2}\right)$$

$r_m = 3.84\times10^8$ m

$R_e = 6.37\times10^6$ m

moon acceleration should be

$$\left(9.8\,\frac{m}{s^2}\right)\cdot\left(\frac{6.37\times10^6}{3.84\times10^8}\right)^2$$

$$= 2.70\times10^{-3}\ m/s^2$$

recall $a = \omega^2 r_m$ and $T = \frac{2\pi}{\omega}$

$$a \doteq \frac{4\pi^2 r_m}{T^2}$$

$$T = 2\pi\sqrt{\frac{r_m}{a}} = (2\pi)\sqrt{\frac{3.84\times10^8\,m}{2.70\times10^{-3}\,m/s^2}} = 2.37\times10^6\ s = 27.4\ \text{days}$$

Kepler's laws and motion of planets

By analyzing data Kepler came up with 3 laws that summarize planetary motion:

1) All planets move in elliptical orbits with the sun at one focus

2) "Equal areas in equal times" Radius vector drawn from sun to planet sweeps out equal areas in equal time intervals.

3. Square of orbit period is proportional to cube of semimajor axis of elliptical orbit