Archimedes

Bouyancy

Body in fluid

fluid exerts upward force = weight of flud displaced by body

Why:

Fluid

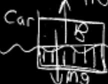
Fluid in equilibrium forces balance

remove volume of fluid, rest of forces remain was in equilibrium ⇒ net upward force = weight

Car in water:

mass = 1000 kg

Volume = 4.0 m^3



How much of car is submerged

Equilibrium: Floats. $\Sigma F_y = 0$

$= B - mg \Rightarrow B = M_{car} g$

Archi: B = weight of water displaced

$B = M_{water\,displ}\, g$, Density $\rho = \frac{M}{V}$

$M_{water} = \rho_{water} V_{water\,displaced}$

$B = \rho_{water} V_{disp}\, g$

$V_{disp} = V_{car\,submerged}$

$\frac{V_{car\,sub.}}{V_{car\,total}}$?

$M_{car} g = \rho_{water} V_{sub} g$

$V_{sub} = \frac{M_{car}}{\rho_{water}} = \frac{1000\,kg}{1.0 \times 10^3\,kg/m^3}$

$\frac{V_{sub}}{V_{TOTAL}} = \frac{1.0\,m^3}{4.0\,m^3} = \frac{1}{4}$ is submerged

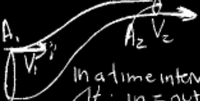
Fluid Dynamics: Fluid Flow

Ideal Fluid ⇒ no viscosity = no internal friction

Fluid Flowing in a pipe (closed)

Mass in = Mass out

cross section area A,
Fluid velocity V
at entrance (1), exit (2)



In a time interval Δt: in = out

Mass in/out = $\rho \cdot$ Volume in/out

in = $\rho A_1 V_1 \Delta t = \rho A_2 V_2 \Delta t$ = out

$A_1 V_1 = A_2 V_2$ Equation of Continuity

Product AV is constant

$A > A'$; $V' > V$

Relation between Pressure and fluid Velocity
Bernoulli's Equation

Fluid at 2 points with
Pressure P, speed V, depth Y

$$P_1 - P_2 = \frac{1}{2}\rho(V_2^2 - V_1^2) + \rho g(Y_2 - Y_1)$$

$$P_1 + \rho g Y_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g Y_2 + \frac{1}{2}\rho V_2^2$$

$$P + \rho g Y + \frac{1}{2}\rho V^2 = \text{Constant}$$

Warning on units.
P = Absolute Pressure, must be
Converted from Gauge Pressure

Pressure Gauge reads
difference between
a system and
Atmospheric
Pressure
= Gauge Pressure

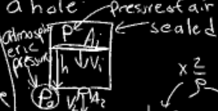
Absolute Pressure
= Gauge Pressure +
Atmospheric Pressure

Density in kg/m^3
Velocity in m/s

Apply to specific cases
Hydrostatics: $V_1 = V_2 = 0$

$$P_1 - P_2 = \rho g(Y_2 - Y_1)$$

Tank of Water with
a hole



$\times \frac{2}{\rho}$

$$P + \frac{1}{2}\rho V_1^2 + \rho g h = P_a + \frac{1}{2}\rho V_2^2$$

$$V_2^2 = V_1^2 + 2\left(\frac{P - P_a}{\rho}\right) + 2gh$$

eq'n of continuity: $V_2 = \frac{A_1}{A_2} V_1 \rightarrow$ Solution

Suppose tank is open to air: $P = P_a$ (atmospheric)

Suppose tank is Large $\Rightarrow$ neglect V_1

$V_2^2 = V_1^2 + \frac{2(P_a - P_a)}{\rho} + 2gh$ (with $V_1^2 \to 0$ and $\frac{2(P_a - P_a)}{\rho} \to 0$)

$V_2 = \sqrt{2gh}$ Torcelli's Theorem

Vessel is Large (V_1 = negligible)

Vessel is sealed ($P \neq P_a$)

Pressure inside so Large that depth (h) neglected

$V_2^2 = V_1^2 + \frac{2(P - P_a)}{\rho} + 2gh$ (with $V_1^2 \to 0$ and $2gh \to 0$)

$V_2 = \sqrt{\frac{2(P - P_a)}{\rho}}$ Rocket Engine

If A = area of exhaust orfice with fluid density ρ, speed V massmflow out in Δt is $\rho A V \Delta t$, momentum $= mV = \rho A V^2 \Delta t$ and $\frac{\Delta P}{\Delta t} = \rho A V^2$

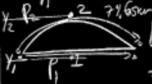
$\frac{\Delta P}{\Delta t} = F \Rightarrow F = \rho A V^2$

$V = \sqrt{\frac{2(P - P_a)}{\rho}} \Rightarrow F = \rho A \left(\frac{2(P - P_a)}{\rho} \right)$

$F = 2A(P - P_a)$

Lift of airplane wing

= Force of 1000N per square of wing area



Need:

Lift = $(P_2 - P_1)$

$= 1000 N/m^2$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho V_2^2$$

Assume $Y_2 - Y_1 = 1$ meter ignore

$$P_1 - P_2 = \frac{1}{2}\rho(V_2^2 - V_1^2)$$

$$V_2^2 = V_1^2 + \frac{2(P_1 - P_2)}{\rho}$$

$$V_2 = \sqrt{V_1^2 + \frac{2(P_1 - P_2)}{\rho}}$$

100m/s = speed = V_1

$$V_2 = \sqrt{(100 m/s)^2 + \frac{2(1000 N/m^2)}{1.3 kg/m^3}}$$

$= 107 m/s$

Top of wing must be 7% Longer in path than bottom