February 19, 2003 Physics 202

EXAM 1

Print your name and section **clearly** **on all** **five** **pages.** (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

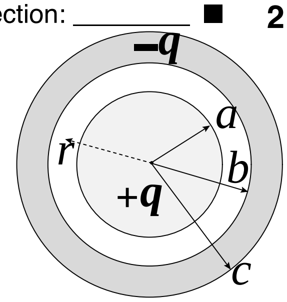
SOLUTION KEY

TOTAL: __________

Possibly useful information:

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$
$k = 8.99 \times 10^{9}\ N\ m^2\ C^{-2}$
$e = 1.602 \times 10^{-19}\ C$

PROBLEM 1



A non-conducting solid sphere of radius ***a*** has total charge ***+q*** uniformly distributed through its volume and is concentric with a larger conducting spherical shell of inner radius ***b***, outer radius ***c***, and total charge ***-q***.

a. What is the electric field as a function of charge ***q*** and distance ***r*** from the common center of the sphere and shell for ***r < a***? *(5 pts.)*

$$\int \vec{E} \cdot d\vec{a} = 4\pi r^2 = \frac{Q}{\varepsilon_0} = \frac{q}{\varepsilon_0}\left(\frac{r}{a}\right)^3 \Rightarrow E = \frac{qr}{4\pi\varepsilon_0 a^3} = \frac{kqr}{a^3}$$

$qr/4\pi\varepsilon_0 a^3$

b. For ***a < r < b*** (state why)? *(5 pts.)*

$4\pi r^2 E = q/\varepsilon_0 \Rightarrow E = q/4\pi\varepsilon_0 r^2 = kq/r^2$
(all charge acts like it is at the center)

$q/4\pi\varepsilon_0 r^2$

c. For ***b < r < c*** (state why)? *(5 pts.)*

E(inside a conductor) = 0

0

d. For ***c < r*** (state why)? *(5 pts.)*

$4\pi^2 E = 0 \Rightarrow E = 0$

0

e. What are the charges on the inner and outer surface of the shell? *(5 pts.)*

$E = 0 \Rightarrow q + Q_i = 0 \Rightarrow Q_i = -q$

$Q_i + Q_O = -q \Rightarrow Q_O = 0$

Inner: -q	Outer: 0

PROBLEM 2

Two parallel conducting metal plates each have area 225 cm^2, separation 0.50 cm (in air) and are connected to a power supply that keeps one at + 0.25 V and the other at 0.00 V. (Ignore edge effects, i.e. assume the field is zero outside).

a. What is the electric field between the plates? *(5 pts.)*

$E = \Delta V/\ell = 0.25\text{ V} / 0.0050\text{ m} = 50.\text{ V/m}$

50. V/m

b. What is the total charge on each plate? *(5 pts.)*

$E = 50\text{ V/m} = \sigma/\varepsilon_0 \Rightarrow \sigma = 50.\ \varepsilon_0\text{ V/m} = 50.\text{ V/m} \cdot (8.85\times10^{-12}\text{ C}^2/\text{Nm}^2) = 4.4\times10^{-10}\text{ C/m}^2$

$Q = \sigma A = (4.4\times10^{-10}\text{ C/m}^2)\ (0.0225\text{ m}^2) = 9.9\times10^{-12}\text{ C}$

9.9×10^{-12} C

c. What is the total energy stored in the electric field? *(5 pts.)*

$U = (\varepsilon_0 E^2/2)(\text{Volume}) = \varepsilon_0 E^2 A\ell/2$

$= (1/2)\ (8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)(50.\text{V/m})^2(0.0225\text{ m}^2)(0.005\text{ m}) = 1.2\times10^{-12}\text{ J}$

or $U = (1/2)QV = (1/2)(9.9\times10^{-12}\text{ C})(0.25\text{ V}) = 1.2\times10^{-12}\text{ J}$

1.2×10^{-12} J

d. How far from the +0.25 V plate is the equipotential surface at 0.10 V? *(5 pts.)*

$\Delta V = Ed \Rightarrow d = \Delta V/E = (0.15\text{ V})/(50.\text{ V/m}) = 0.0030\text{ m} = 0.30\text{ cm}$

0.30 cm

e. What is the work in Joules to move one electron from the 0.00 V plate to the other? *(5 pts.)*

$W = Ve = (0.25\text{ V})(1.602\times10^{-19}\text{ C}) = 4.0\times10^{-20}\text{ J}$

4.0×10^{-20} J

PROBLEM 3

A parallel plate capacitor with area of each plate 20.0 cm^2 and plate separation 4.0 mm is filled with air (dielectric strength 3.0×10^6 V/m).

a. What is the capacitance? *(5 pts.)*

$$C = \frac{(8.85\times10^{-12}\,\mathrm{F/m})(2.00\times10^{-3}\,\mathrm{m}^2)}{4.0\times10^{-3}\,\mathrm{m}} = 4.4\times10^{-12}\,\mathrm{F} = 4.4\,\mathrm{pF}$$

4.4 pF

b. What is the maximum charge the capacitor can hold? *(5 pts.)*

$V_{max} = E_{max}\cdot d = (3.0 \times 10^6\ \mathrm{V/m})(4.0\times10^{-3}\ \mathrm{m}) = 1.2\times10^4\ \mathrm{V}$
$Q_{max} = CV_{max} = (4.4\times10^{-12}\ \mathrm{F})(1.2\times10^4\ \mathrm{V}) = 5.3\times10^{-8}\ \mathrm{C}$

53 nC

c. After a battery charges the capacitor to 1.0×10^{-9} C and is disconnected, a Teflon ($\kappa = 2.1$) sheet 2.0 mm thick is inserted between and parallel to the plates. What is the electric field in the air and inside the Teflon? *(5 pts.)*

$$E_{air} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} = \frac{1.0\times10^{-9}\,\mathrm{C}}{(8.85\times10^{-12}\,\mathrm{F/m})(2.0\times10^{-3}\,\mathrm{m}^2)} = 5.6\times10^4\,\frac{\mathrm{V}}{\mathrm{m}}$$

$$E_{teflon} = \frac{\sigma}{\varepsilon} = \frac{Q}{\kappa\varepsilon_0 A} = \frac{E_0}{\kappa} = \frac{5.6\times10^4\,\mathrm{V/m}}{2.1} = 2.7\times10^4\,\frac{\mathrm{V}}{\mathrm{m}}$$

Air: 56 kV/m	Teflon: 27 kV/m

d. What is the total voltage drop now across the plates? *(5 pts.)*

$V = (E_0 \times 2.0\ \mathrm{mm}) + (E_{Teflon} \times 2.0\ \mathrm{mm}) = (5.6 \times 10^4\ \mathrm{V/m})(2.0 \times 10^{-3}\ \mathrm{m}) + (2.7 \times 10^4\ \mathrm{V/m})(2.0 \times 10^{-3}\ \mathrm{m}) = 112\ \mathrm{V} + 54\ \mathrm{V} = 166\ \mathrm{V}$

170 V

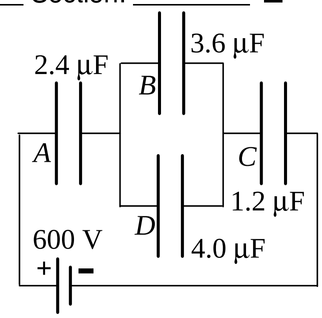
e. What is the new capacitance? *(5 pts.)*

$C = Q/V = (1.0 \times 10^{-9}\ \mathrm{C})/(166\ \mathrm{V}) = 6.0 \times 10^{-12}\ \mathrm{F}$

6.0 pF

PROBLEM 4

4 capacitors, A, B, C, and D, with capacitances 2.4 μF, 3.6 μF, 1.2 μF, and 4.0 μF, respectively, are charged up with a 600.0 V battery as shown and then the battery is disconnected



a. What is the total capacitance of the 4 capacitors combined? *(5 pts.)*

$$C' = C_B + C_D = 3.6\,\mu F + 4.0\,\mu F = 7.6\,\mu F$$

$$\frac{1}{C} = \frac{1}{C_A} + \frac{1}{C'} + \frac{1}{C_C} = \frac{1}{2.4\,\mu F} + \frac{1}{7.6\,\mu F} + \frac{1}{4.2\,\mu F} = \frac{1}{0.72\,\mu F}$$

0.72 µF

b. What is the charge stored on capacitors A and C? *(5 pts.)*

$Q_A = Q_C = CV = (0.72 \times 10^{-6}\ F)(600.0\ V) = 4.3\times 10^{-4}\ C$

A: 0.43 mC	C: 0.43 mC

c. What is the voltage across capacitors A and C? *(5 pts.)*

$V_A = Q/C_A = (4.3 \times 10^{-4}\ C)/(2.4 \times 10^{-6}\ F) = 1.8 \times 10^2\ V$
$V_C = Q/C_C = (4.3 \times 10^{-4}\ C)/(1.2 \times 10^{-6}\ F) = 3.6 \times 10^2\ V$

A: 180 V	C: 360 V

d. What is the voltage across capacitors B and D? *(5 pts.)*

$V_B = V_D = Q/C' = (4.3 \times 10^{-4}\ C)/(7.6 \times 10^{-6}\ F) = 57\ V$

B: 57 V	D: 57 V

e. What is the total energy stored in the 4 capacitors? *(5 pts.)*

$U = (1/2)CV^2 = (1/2)(0.72 \times 10^{-6}\ F)(600.0\ V)^2 = 0.13\ J$

0.13 J